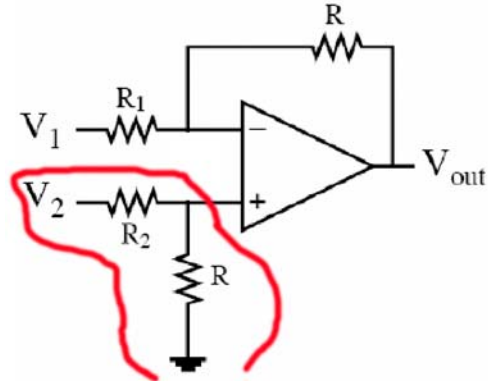


If the op amp is operating correctly, then $V_+ = V_-$. There is no potential difference between the two sides of the resistor that V_+ and V_- . Therefore, no current flows, and we can redraw the circuit as shown below.



First consider only the part of the circuit circled in red. We can write two equations:

$$V_2 - R_2 I = V_+ \quad \text{Equation 1}$$

$$V_+ - R I = 0 \quad \text{Equation 2}$$

In these equations, there is only one current, I , because the resistors are in series. None of the current run into the opamp because it is ideal, and it therefore has infinite input impedance.

Rewriting Equation 2 to solve for I , we obtain:

$$I = V_+ / R \quad \text{Equation 3}$$

Substituting Equation 3 into Equation 1 yields:

$$V_2 - R_2(V_+ / R) = V_+ \quad \text{Equation 4}$$

Then solve for V_+ as follows:

$$V_2 - (R_2/R)V_+ = V_+$$

$$V_2 = V_+ + (R_2/R)V_+$$

$$V_2 = V_+(1 + (R_2/R))$$

$$V_+ = V_2 \left(\frac{R}{R + R_2} \right) \quad \text{Equation 5}$$

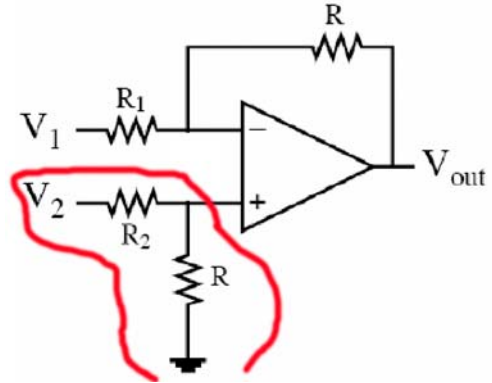
Now consider the part of the circuit that is not circled in red.

We can again write two equations.

$$V_1 - R_1 I = V_- \quad \text{equation 6}$$

$$V_- - R I = V_{out} \quad \text{equation 7}$$

As before, there is only one current, I, (different from the previous current I), because no current runs into the ideal op-amp.



We now solve equation 6 for I:

$$I = (V_1 - V_-) / R_1 \quad \text{equation 8}$$

And substitute that expression for I into equation 7:

$$V_- - R[(V_1 - V_-) / R_1] = V_{out} \quad \text{equation 9}$$

Now we want to solve for V₋. First, multiply through by R₁:

$$R_1 V_- - R V_1 + R V_- = V_{out} R_1 \quad \text{equation 10}$$

$$(R_1 + R) V_- = V_{out} R_1 + R V_1 \quad \text{equation 11}$$

$$V_- = \left(\frac{V_{out} R_1 + R V_1}{R + R_1} \right) \quad \text{equation 12}$$

Now notice that equation 5 is an equation for V₊, and equation 12 is an equation for V₋.

But we know that if the op amp is working correctly, then V₊ = V₋.

So we can equate equation 12 and equation 5:

$$V_- = \left(\frac{V_{out} R_1 + R V_1}{R + R_1} \right) = V_+ = V_2 \left(\frac{R}{R + R_2} \right)$$

Solving for V_{out} gives:

$$V_{out} = \left(\frac{R(R_1 + R) V_2}{R_1(R_2 + R)} \right) - \left(\frac{R}{R_1} \right) V_1$$

$$\text{If } R_2 = R_1 \text{ then } V_{out} = \left(\frac{R V_2}{R_1} \right) - \left(\frac{R}{R_1} \right) V_1$$