Nonlinear theory of power transfer between multiple crossed laser beams in a flowing plasma

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Analytic results are obtained for power transfer among crossing, equal frequency, laser beams, each smoothed by a random phase plate, in a flowing homogeneous plasma. For beams with well-separated directions, interbeam coupling transfers power, while intrabeam coupling causes beam deflection. For any pair of such beams, the beam with the largest positive projection on the flow direction will drain power from the other. © *1998 American Institute of Physics.* [S1070-664X(98)01505-5]

I. INTRODUCTION

The transfer of power between crossed laser beams has been a consideration in the design of the large number of such beams that enter hohlraum targets in the National Ignition Facility (NIF): such a transfer would upset the accurate beam pointing that is required to achieve high gain implosions of capsules in inertial confinement fusion.[1] When a pair of beams differ in frequency so as to resonantly drive the sound wave induced by their beat ponderomotive force, efficient transfer from the higher-frequency to the lower-frequency beam has been predicted[2] and observed.[3] To avoid such a coupling, it was thought that the various beams should have the same frequency.

In this paper it will be shown that the presence of plasma flow can induce a significant power transfer between monochromatic beams in the steady state. An analytic theory is presented for the case in which each beam is smoothed by a random phase plate[4] (RPP), and it is validated by simulations in a regime where self-focusing is not negligible. This theory is naturally thought of as representing transfer of power between pairs of beams, interbeam coupling, and the redistribution of power within a beam, intrabeam coupling. The latter is primarily responsible for beam deflection by flow[5–8] when the beam directions are well separated.

II. MODEL EQUATIONS

A simplified model of the superposition of several RPP smoothed laser beams, each labeled by its centroid wave vector, $\mathbf{q}$, is obtained by assigning a static phase, $\phi(\mathbf{k})$, to each Fourier mode of the electric field, $\mathbf{E}$ (in a particular plane, $z=0$, of the optic focal region), which for each $\mathbf{k}=(k_x,k_y)$, is a statistically independent random variable, such that

$$\langle \exp i[\phi(\mathbf{k})-\phi(\mathbf{k}')]\rangle=\delta_{\mathbf{k}\mathbf{k}'}, \tag{1}$$

$$\mathbf{E}=\hat{\mathbf{e}}\ \mathrm{Re}\ \epsilon(\mathbf{x})\exp(-i\omega_0 t), \tag{2}$$

$$\epsilon(\mathbf{x})=\sum_{\mathbf{k}}\hat{\epsilon}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{x})=\sum_{\mathbf{q}}\epsilon(\mathbf{x}|\mathbf{q}), \tag{3}$$

$$\epsilon(\mathbf{x}|\mathbf{q})=\sum_{|\mathbf{k}-\mathbf{q}|<k_m}|\hat{\epsilon}(\mathbf{k})|\exp i[\mathbf{k}\cdot\mathbf{x}+\phi(\mathbf{k})]. \tag{4}$$

Here $\mathbf{q}$ is the projection of each beam's centroid wavevector in the x-y plane, $\hat{\mathbf{e}}$ is a unit vector in the x-y plane, $\mathbf{x}=(x,y)$, and ω_0 is the laser angular frequency. In the top hat model of intensity distribution across the RPP, $|\hat{\epsilon}|$ is a constant in the disk $|\mathbf{k}-\mathbf{q}|\leq k_m=k_0/(2F)$, and vanishes otherwise, where k_0 is the laser wave number and F is the optic "F" number.[9]

The sum over $\mathbf{q}$ is thought of having a modest number of terms. For example, a NIF hohlraum will have 24 beams entering each side. For the purpose of applying this model to a more realistic, finite beam geometry, it is assumed that the simplified model presented above is applicable locally in a region of the plasma if the spatial region in which a particular subset of the 24 beams overlap is large compared to a speckle width—a transverse correlation length of a single beam—about $F\lambda_0$, where λ_0 is the laser wavelength. The sum over $\mathbf{k}$ has a much larger number of terms, with each term corresponding to a particular RPP element.

In the paraxial wave approximation,[10] with polarization coherent light, and for time scales large compared to light transit times,

$$i\frac{\partial}{\partial z}\epsilon=-\frac{1}{2k_0}\Delta\epsilon+\frac{k_0}{2}\frac{n_0}{n_c}\frac{\delta n_e(\mathbf{x},z,t)}{n_0}\epsilon,$$
$$\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2, \tag{5}$$

n_c is the critical density, and n_0 is the background density. Isothermal, linearized hydro, with the normalized Green's function, L, is used to determine the density fluctuation in the steady state,[11]

$$\frac{\delta n(\mathbf{x},z,t)}{n_0}=-\frac{|\langle\delta n\rangle|}{n_0}\int L(\mathbf{x}-\mathbf{x}')\frac{U(\mathbf{x}',z)}{\langle U\rangle}d\mathbf{x}', \tag{6}$$

where $|\langle\delta n\rangle|$ is the magnitude of the ponderomotive density response based upon the average ponderomotive potential, $\langle U\rangle\propto\langle|\epsilon|^2\rangle$, in a quiescent (nonflowing) plasma. In practical

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units, if the ion pressure is negligible, $|\langle \delta n\rangle|/n_0 \approx 0.90 \times 10^{-13}(\lambda_0/\mu\mathrm{m})^2(\mathrm{cm}^2\langle I\rangle/\mathrm{W})(\mathrm{eV}/T_e)$ where T_e is the electron temperature, λ_0 is the laser wavelength, and $\langle I\rangle$ the average laser intensity. In Fourier space,

$$\hat{L}^{-1}(\mathbf{k}) = 1 - (\hat{\mathbf{k}}\cdot\mathbf{M})^2 + 2i\nu_{ia}\hat{\mathbf{k}}\cdot\mathbf{M}. \tag{7}$$

Here $\hat{\mathbf{k}}$ is a unit vector in the direction of $\mathbf{k}$, the dimensionless Landau damping coefficient, ν_{ia}, can vary over a wide range, from about 0.01 to 0.5, and $\mathbf{M}$ is the Mach vector based on the plasma flow, $\mathbf{v}_0$, and the ion acoustic speed, c_s, $\mathbf{M} = \mathbf{v}_0/c_s$. Since collisional absorption has been omitted from Eq. (5), it is assumed that its effect is negligible over the range of z of interest.

Note that Eq. (1) implies that the fluctuations of ϵ are spatially homogeneous in any plane with fixed z, and this property is maintained as ϵ propagates according to Eqs. (5) and (6).

III. GAUSSIAN ANSATZ

An equation describing the spatial development of the energy spectrum, $\langle|\hat{\epsilon}(\mathbf{k},z)|^2\rangle$, is now obtained. As the laser propagates into the plasma, the nonlinear coupling in Eq. (5) will generate phase correlations. Perturbatively[12] these correlations grow at a spatial rate that is proportional to $\langle U\rangle^2$, while the rate of change of the spectrum goes like $\langle U\rangle$, as shown below. Therefore, for small enough $\langle U\rangle$ (small enough average laser intensity), the spectrum may develop significantly before the approximation of statistically independent phases is violated.

If the sum over $\mathbf{k}$ in Eq. (3) involves a large number of terms, then phase independence implies that the fluctuations of ϵ are Gaussian. The ansatz is made that over the range of z of interest, the particular property of Gaussian fields shown below in Eq. (8) is satisfied. Limited numerical tests of this equality will be presented with the simulation results.

Gaussian statistics and Eq. (1) imply that

$$\begin{aligned}\langle \epsilon(\mathbf{x}_1)\epsilon^*(\mathbf{x}_2)\epsilon(\mathbf{x}_3)\epsilon^*(\mathbf{x}_4)\rangle &= \langle \epsilon(\mathbf{x}_1)\epsilon^*(\mathbf{x}_2)\rangle\langle \epsilon(\mathbf{x}_3)\epsilon^*(\mathbf{x}_4)\rangle \\ &\quad + \langle \epsilon(\mathbf{x}_1)\epsilon^*(\mathbf{x}_4)\rangle\langle \epsilon(\mathbf{x}_3)\epsilon^*(\mathbf{x}_2)\rangle,\end{aligned} \tag{8}$$

where, e.g., $\mathbf{x}_1$, is any spatial location ''1.'' It follows from Eqs. (5), (6), and (8) that

$$\begin{aligned} i\frac{\partial}{\partial z} b(\mathbf{x},z) = -\frac{k_0}{2}\frac{n_0}{n_c}\frac{|\langle\delta n\rangle|}{n_0}\int [L(\mathbf{x}-\mathbf{x}') - L(-\mathbf{x}')] \\ \times b(\mathbf{x}-\mathbf{x}',z)b(\mathbf{x}',z)d\mathbf{x}',\end{aligned} \tag{9}$$

where $b(\mathbf{x},z)$ is the normalized two-point electric field amplitude correlation function,

$$b(\mathbf{x},z) = \langle \epsilon(\mathbf{x},z)\epsilon^*(\mathbf{0},z)\rangle/\langle|\epsilon|^2\rangle, \tag{10}$$

or in Fourier space, $\hat{b}(\mathbf{k},z) = \langle|\hat{\epsilon}(\mathbf{k},z)|^2\rangle/\langle|\epsilon|^2\rangle$, approximating Fourier sums by integrals,

$$\begin{aligned}\frac{\partial}{\partial z}\ln \hat{b}(\mathbf{k},z) = k_0\frac{n_0}{n_c}\frac{|\langle\delta n\rangle|}{n_0} \\ \times\int \frac{d\mathbf{p}}{(2\pi)^2}\hat{b}(\mathbf{k}-\mathbf{p},z)G(\hat{\mathbf{p}}\cdot\mathbf{M}),\end{aligned} \tag{11}$$

with

$$G(u) = \frac{2\nu_{ia}u}{(1-u^2)^2 + 4\nu_{ia}^2u^2}. \tag{12}$$

Since for zero spatial separation, b is normalized to unity,

$$\int \frac{d\mathbf{p}}{(2\pi)^2}\hat{b}(\mathbf{p},z) = 1. \tag{13}$$

Equation (9) is consistent with the conservation of laser power, $(d/dz)\int|\epsilon|^2\, d\mathbf{x} = 0$, as can be seen by evaluating it for $\mathbf{x} = 0$ to obtain an equation for $(d/dz)\langle|\epsilon|^2\rangle$, and noting that the integrand vanishes.

A curious feature of Eq. (9) is that its derivation is independent of the presence of the diffraction term in Eq. (5). The *validity* of Eq. (9), however, does depend on diffraction because it is one of the players in determining the importance of self-focusing, which, if significant, invalidates the Gaussian ansatz.

IV. POWER TRANSFER AND BEAM DEFLECTION

Corresponding to the beam decomposition of Eq. (3), one has

$$\hat{b}(\mathbf{k},z) = \sum_{\mathbf{q}} \hat{b}(\mathbf{k},z|\mathbf{q}). \tag{14}$$

It is assumed that the beam directions are separated so that any given Fourier mode, $\mathbf{k}$, belongs to, at most, one beam, say beam ''$\mathbf{q}$.'' Substitute Eq. (14) into Eq. (11),

$$\frac{\partial}{\partial z}\hat{b}(\mathbf{k},z|\mathbf{q}) = k_0\frac{n_0}{n_c}\frac{|\langle\delta n\rangle|}{n_0}\sum_{\mathbf{q}'}\gamma(\mathbf{k},z|\mathbf{q}')\hat{b}(\mathbf{k},z|\mathbf{q}), \tag{15}$$

with the dimensionless gain coefficient, $\gamma(\mathbf{k},z|\mathbf{q}')$, given by

$$\gamma(\mathbf{k},z|\mathbf{q}') = \int \frac{d\mathbf{p}}{(2\pi)^2}\hat{b}(\mathbf{k}-\mathbf{p},z|\mathbf{q}')G(\hat{\mathbf{p}}\cdot\mathbf{M}). \tag{16}$$

The terms with $\mathbf{q}\neq\mathbf{q}'$ represent interbeam power transfer, and the term with $\mathbf{q}=\mathbf{q}'$, redistributes power within beam $\mathbf{q}$.

A. Power transfer

Consider a particular pair of beams, $\mathbf{q}$ and $\mathbf{q}'$. The vector $\mathbf{p}$ in Eq. (16) may be thought of as pointing from the element of one beam to an element of the other. If the beams are separated enough so that the projection of $\mathbf{p}$ on $\mathbf{M}$ does not change sign as the various beam elements are considered, then the beam that points upstream with respect to the other, i.e., the vector from its direction (the location of its centroid in Fourier space) to the other beam's direction has a positive projection along the plasma flow, will lose power due to this particular coupling.

To gain a qualitative measure of the power transfer rate, assume (as is the case for the top hat model at $z=0$) that $\hat{b}(\mathbf{k},z|\mathbf{q})$ is a constant on the right-hand side (RHS) of Eqs. (15) and (16), e.g., for $|\mathbf{k}-\mathbf{q}|<k_m$, $\hat{b}(\mathbf{k},z|\mathbf{q})/(2\pi)^2 = P(\mathbf{q},z)/(\pi k_m^2)$, where, in general, $P(\mathbf{q},z)$ is the fractional power in each beam,

$$P(\mathbf{q},z)=\int \frac{d\mathbf{k}}{(2\pi)^2}\,\hat{b}(\mathbf{k},z|\mathbf{q}),\quad \sum_{\mathbf{q}} P(\mathbf{q},z)=1. \tag{17}$$

Integrate Eq. (15) over $\mathbf{k}$ for $|\mathbf{k}-\mathbf{q}|<k_m$ to obtain

$$\begin{aligned}\frac{d}{dz}\ln P(\mathbf{q},z) &= k_0\,\frac{n_0}{n_c}\,\frac{|\langle\delta n\rangle|}{n_0}\sum_{\mathbf{p}=\mathbf{q}-\mathbf{q}'\neq 0}\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia})\\ &\quad\times P(\mathbf{q}',z),\end{aligned} \tag{18}$$

with

$$\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia}) = \frac{1}{\pi^2}\int_{|\mathbf{k}-\mathbf{p}/k_m|<1} d\mathbf{k}\int_{|\mathbf{k}-\mathbf{k}'|<1} d\mathbf{k}'\,\mathbf{G}(\hat{\mathbf{k}}'\cdot\mathbf{M}). \tag{19}$$

Since $G(u)=-G(-u)$, $\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia})=-\Gamma(-\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia})$, so that power is conserved in the transfer between any two beams.

1. Very well-separated beams

The assumption of separate beams implies that $p/k_m>2$ (recall that $\mathbf{p}$ is the wave vector separation between a pair of beams, $\mathbf{q}$ and $\mathbf{q}'$, $\mathbf{p}=\mathbf{q}-\mathbf{q}'$). If $p/k_m\gg 1$, then the angle between $\mathbf{k}'$ and $\mathbf{M}$ in Eq. (19), θ, does not change much as the $\mathbf{k}$ and $\mathbf{k}'$ range over their domains of integration, $\delta\theta\approx 2k_m/p$. If, in turn, the corresponding change in G is small, $\delta G/G\ll 1$, then

$$\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia})\approx G(\hat{\mathbf{p}}\cdot\mathbf{M}). \tag{20}$$

Given that $\delta\theta\ll 1$, there are various regimes in which it is simple to estimate when $\delta G/G\ll 1$. Let $\hat{\mathbf{k}}'\cdot\mathbf{M}=M\cos\theta$, so that $\delta(\hat{\mathbf{k}}'\cdot\mathbf{M})=\hat{\mathbf{p}}\cdot\mathbf{M}\,\delta\theta\tan\theta$. For example, if $(\hat{\mathbf{p}}\cdot\mathbf{M})^2\ll 1$, then $\delta G/G\approx\delta\theta\tan\theta$, while if $|\hat{\mathbf{p}}\cdot\mathbf{M}|\approx 1$ and $|1-|\hat{\mathbf{p}}\cdot\mathbf{M}||>\nu_{ia}$, then $\delta G/G\approx 2\,\delta\theta\tan\theta/|1-|\hat{\mathbf{p}}\cdot\mathbf{M}||$.

If there is only a single finite intensity beam, then the gain rate implied by Eqs. (18) and (20) for a fluctuation well separated by wave vector $\mathbf{p}$ from the beam center is given by $k_0(|\langle\delta n\rangle|/n_c)G(\hat{\mathbf{p}}\cdot\mathbf{M})$, which is identical[13] to that obtained for the self-focusing instability of a coherent pump in a flowing plasma, in the limit where the wave number of the perturbation is large compared to that of the marginally stable mode in the absence of flow, i.e., $p^2\gg k_0^2|\langle\delta n\rangle|/n_0$.

2. A numerical example

In general, the integrals in Eq. (19) require numerical evaluation. The following particular cases are now considered:, $p/k_m=3$ and $\nu_{ia}=0.2$, with either $\beta\equiv\cos^{-1}(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}})=0$ or $\beta=\pi/4$. Figure 1 shows the dependence of Γ on M, compared with that of G. In this regime of relatively strong acoustic damping, the agreement is quite good in the subsonic regime for both cases, and for all Mach numbers when $\beta=0$. Since the accuracy of this approximation improves with increasing p/k_m there will also be excellent agreement for the case $p/k_m=4$, which is considered later in Sec. V.

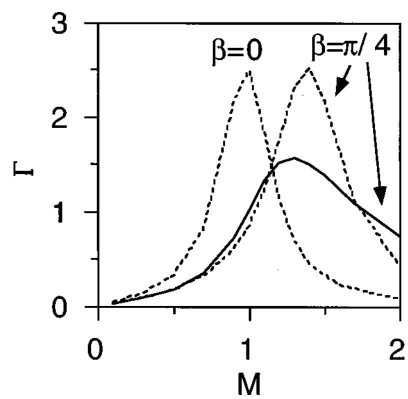

FIG. 1. Variation of the normalized two-beam power transfer rate, Γ, with Mach number, M, for the case $\beta\equiv\cos^{-1}(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}})=0$, i.e., the flow is parallel to the beam separation direction, and the case of $\beta=\pi/4$. The solid curve is obtained from Eq. (19), and the dashed curves are given by G, the simple estimate of Eq. (20). For $\beta=0$ only G is shown since Γ is numerically determined to be very well approximated by G for this case.

3. The regime of small acoustic damping

For very small values of ν_{ia}, $G(u)$ depends very sensitively on its argument when $u\approx 1$, and the approximation of Eq. (20) may be poor. In this section, with the aid of a simpler model than the top hat spectrum, an analytic result is again obtained for Γ.

Since the four-dimensional integral in Eq. (19) is difficult to study numerically when G has a very narrow resonance, an analytically simpler model is now considered. Instead of the top hat model for a single beam's spectrum, take it as a Gaussian, so that instead of Eq. (19), one has, after a change of variable of integration,

$$\begin{aligned}\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia}) &= \frac{1}{4\pi^2}\int\int d\mathbf{k}\,d\mathbf{k}'\,G(\hat{\mathbf{k}}\cdot\mathbf{M})\\ &\quad\times\exp\left(-\frac{(\mathbf{k}+\mathbf{k}'-\mathbf{p}/k_m)^2+\mathbf{k}'^2}{2}\right)\\ &= \frac{1}{4\pi}\int d\mathbf{k}\,G(\hat{\mathbf{k}}\cdot\mathbf{M})\\ &\quad\times\exp\left(-\frac{(\mathbf{k}-\mathbf{p}/k_m)^2}{4}\right).\end{aligned} \tag{21}$$

If the beams are well separated, then in the limit $\nu_{ia}\to 0$, only the resonance at, e.g., $\hat{\mathbf{k}}\cdot\mathbf{M}=1$, is encountered, and $G(\hat{\mathbf{k}}\cdot\mathbf{M})\to(\pi/2)\,\delta(x-1)$, with $x=M\cos\theta$, and θ is the angle between $\mathbf{k}$ and $\mathbf{M}$. It follows that

$$\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia}\to 0)$$
$$=\frac{1}{8\sqrt{M^2-1}}\exp\left(-\frac{p^2}{4k_m^2}\right)$$
$$\times\int_0^\infty dkk\ \exp\left(\frac{2k(p/k_m)\cos(\beta-\theta)-k^2}{4}\right), \qquad (22)$$

where $\cos\theta=1/M$.

The integrand and therefore Γ is a maximum when $\theta=\beta$, i.e., when the vector joining the beam centers (in Fourier space), is on the Mach cone. If

$$(p/k_m)\cos(\beta-\theta)\gg 1, \qquad (23)$$

then the integrand strongly peaks at $k\approx(p/k_m)\cos(\beta-\theta)$ with the value $(p/k_m)\cos(\beta-\theta)\exp[(p/2k_m)^2\cos^2(\beta-\theta)]$, so that by the method of steepest descent,

$$\Gamma(\hat{\mathbf{p}}\cdot\hat{\mathbf{M}},M,p/k_m,\nu_{ia}\to 0)$$
$$=\frac{\sqrt{\pi}}{4\sqrt{M^2-1}}(p/k_m)\cos(\beta-\theta)$$
$$\times\exp\left[-\left(\frac{p}{2k_m}\right)^2[1-\cos^2(\beta-\theta)]\right]. \qquad (24)$$

In fact, numerical evaluation of Eq. (22) shows that there is agreement with Eq. (24) to within a few percent for $(p/k_m)\cos(\beta-\theta)$ as small as 2.

For $\cos^2(\beta-\theta)\neq 1$, Γ increases with p/k_m until it reaches a maximum at $(p/k_m)^2=2/[1-\cos^2(\beta-\theta)]$. In order that this maximum is consistent with the ordering of Eq. (23), e.g., $(p/k_m)\cos(\beta-\theta)>2$, it is necessary that $|\beta-\theta|<0.6$. When $\theta=\beta$, Γ keeps increasing with p/k_m because more modes are closer to being resonant. Otherwise, Γ eventually decreases with increasing p/k_m because the beam centers are off the Mach cone by a finite angle, and once $\delta\theta\approx 2k_m/p$ exceeds this angle, fewer and fewer modes are resonant. If the top hat spectrum were retained, instead of the Gaussian, then there would be a sharp cutoff of Γ at $2k_m/p\approx|\beta-\theta|$.

B. Beam deflection for well-separated beams

Define the propagation direction of beam q by its wave vector centroid,

$$\langle\mathbf{q}(z)\rangle\equiv\frac{\int(dk_xdk_y/(2\pi)^2)\mathbf{k}\hat{b}(\mathbf{k},z|\mathbf{q})}{\int(dk_xdk_y/(2\pi)^2)\hat{b}(\mathbf{k},z|\mathbf{q})}. \qquad (25)$$

In general, its evolution depends upon coupling to all the other beams. If the beams are very well separated, then interbeam couplings simply increase or decrease the power in beam $\mathbf{q}$, but not the distribution of power within that beam, so that there is no direct contribution to $d\langle\mathbf{q}(z)\rangle/dz$. Self-coupling the $\mathbf{q}=\mathbf{q}'$ term in Eq. (15) redistributes that beam's power, leading to a finite value of $d\langle\mathbf{q}(z)\rangle/dz$. If just this coupling is retained to obtain the self-induced rate of change of the beam direction, it can be shown that this is identical to that previously obtained for a single beam.[7,14]

If there are many well-separated beams, then power transfer will dominate beam deflection because the former is a cumulative effect, i.e., the most downstream pointing of all the beams receives power from all the other beams, while beam deflection is determined by single beam intensities.[15]

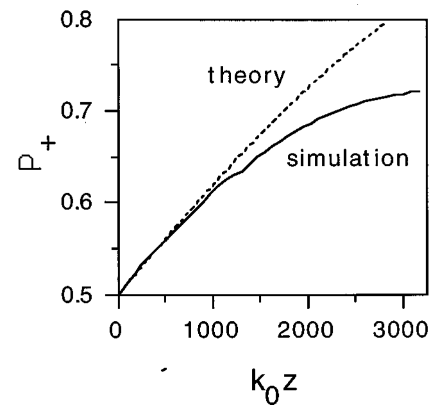

FIG. 2. Spatial development of power in the upstream member of a pair of RPP beams: the solid curve is the simulation; the dashed curve is the solution to Eq. (26).

V. SIMULATION VERSUS THEORY FOR TWO CROSSING BEAMS

The numerical solution, Eqs. (5) and (6), is compared with the statistical theory given by Eq. (18), for the following particular two-beam case, whose parameters are chosen to qualitatively be in the regime for the crossing of a pair of beams of the inner and outer cone of beams entering one side of a NIF[1] hohlraum: $T_e=3$ keV, $n_0/n_c=0.1$, $\nu_{ia}=0.2$, $M=0.5$, $\lambda_0=0.35\,\mu$m, $F=8$ single beam optic, overlapped average laser intensity$=\langle I\rangle=4$E15 W/cm^2, $p/k_m=4$ and[16] $\hat{\mathbf{p}}\cdot\hat{\mathbf{M}}=1$. These parameters imply that $|\langle\delta n\rangle|/n_0\approx 0.015$, while in Sec. IV A we imply that G is a good approximation to Γ, so that $\Gamma\approx 0.33$. Numerical parameters are the following: number of grid points in the x and y directions, $n_x=n_y=128$, periodicity length in the x and y directions, $L_x=L_y\approx 129\lambda_0$, and step size in the z direction, $dz\approx 7.5/k_0$. A standard split step method is used to advance Eq. (5), with the diffraction term evaluated in Fourier space.

Let P_+ be the fractional power in the upstream beam. For these parameters, Eq. (18) implies that

$$dP_+/d(k_0z)\approx 4.83\times 10^{-4}P_+(1-P_+). \qquad (26)$$

The boundary condition consists of independent phases for the two RPP beams, with the power equally shared between them,[17] so that $P_+(z=0)=0.5$. The solution of Eq. (26) is

$$4.83\times 10^{-4}k_0z=\ln[P_+/(1-P_+)]. \qquad (27)$$

This result is compared with the simulation in Fig. 2. The agreement must be good near $z=0$ since the boundary condition ensures that the fluctuations there are nearly Gaussian, and non-Gaussian correlations develop over a finite distance, as evidenced by Fig. 3, where the flatness (dashed curve), $S=\langle|\epsilon|^4\rangle/\langle|\epsilon|^2\rangle^2$ is shown (all factors of ϵ in this expression are evaluated at a given spatial location) and the ensemble average has been approximated by spatial average in the x-y plane. If the Gaussian ansatz were valid, then Eq. (8) implies that $S=2$. The deviation of S from this value at $z=0$ is a

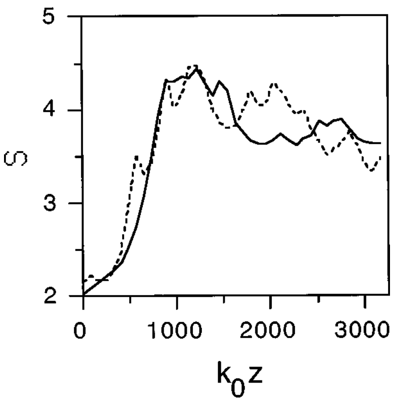

FIG. 3. Dashed curve: the growth of flatness for the case of Fig. 2. Solid curve: flatness for the system that has twice the linear dimension.

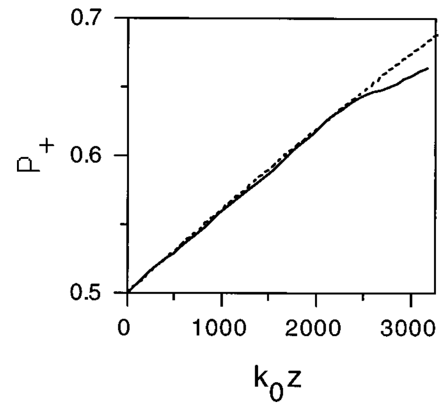

FIG. 4. The same as Fig. 2, but with half the laser intensity.

measure of the corrections to Gaussian behavior owing to the finite system size, e.g., the number of RPP elements in each beam is ≈ 200, while strictly Gaussian statistics are obtained only in the limit of an infinite number of elements. The solid curve shows the flatness from a simulation with twice the linear dimension. It is closer to Gaussian at $z=0$, consistent with there being a larger number of elements per beam, ≈ 800, and it is a smoother curve, suggesting that the finer scale bumps in the dashed curve are fluctuations and not representative of the ensemble average. There was only a few percent change in $P_+(z)$ between the two systems.

Note that only after a relatively short propagation distance, say at $z=1700/k_0=95\ \mu$m, one beam has about twice the power of the other, initially equal powered, beam.

The growth of S as the beam develops in space may be interpreted as a self-focusing effect. At $z=0$, the figure of merit[18] for self-focusing, $\langle P_{\rm hs}\rangle/P_c \approx 0.20$, where $\langle P_{\rm hs}\rangle$ is the power in a single beam's hot spot whose nominal intensity is the single beams' average, and P_c is the critical power for explosive ponderomotive self-focusing. Since typical hot spots have a peak intensity of about three times the average,[18,19] self-focusing is expected to be non-negligible. With regard to self-induced beam deflection, Fig. 15 of Ref. 5 shows that self-focusing is expected to have a large effect[20] in this regime. The measured deflection of the upstream beam, over the range of the z axis simulated, is, in fact, about ten times the estimate provided by the theory based on non-self-focused beams that have a top hat spectrum.[7]

In Fig. 4, the simulated power transfer for the weaker case $\langle I\rangle = 2$E15 W/cm^2, is in much better agreement with the theoretical result [obtained from Eq. (27) by the replacement $4.83 \rightarrow 2.42$]. To an excellent approximation, the same simulation result is obtained by maintaining the intensity at 4E15 W/cm^2, while reducing n_0/n_c to 0.05, so that $\langle P_{\rm hs}\rangle/P_c$ is the same for these two weaker cases. This is expected if, as apparently is the case, self-focusing events that are strongly limited by density depletion effects are not significant.[21]

In general, one does not expect good agreement between the simple theory and the model equations, Eqs. (5) and (6), when self-focusing is significant for any single beam. However, for well-separated beams, diffraction effects are more significant (and self-focusing effects less significant) for interbeam couplings because the wave numbers probed by such couplings, p, by assumption are much larger than those wave numbers probed by intrabeam coupling, which are of magnitude k_m. Perhaps this is why in the above simulations the power transfer rate was more accurately predicted than the beam deflection rate. Since these wave numbers are associated with acoustic fluctuations, interbeam couplings have associated acoustic frequencies that are also larger and hence less susceptible to temporal smoothing than intrabeam couplings. It is therefore possible that a given amount of temporal bandwidth (either externally imposed or self-induced) is at once both adequate to suppress self-focusing and inadequate to suppress interbeam power transfer.

VI. MITIGATING CIRCUMSTANCES

There are various mechanisms that may reduce the interbeam power transfer. These include polarization, frequency shifts between the beams, and plasma inhomogeneity.

A. Polarization

If each beam is polarization coherent, but for a given pair of beams, $\mathbf{q}$ and $\mathbf{q}'$, the polarization vectors $\hat{\mathbf{e}}_{\mathbf{q}}$ and $\hat{\mathbf{e}}_{\mathbf{q}'}$ may be distinct, then it can be shown that Eqs. (15) and (16) are replaced by

$$\frac{\partial}{\partial z}\,\hat{b}(\mathbf{k},z|\mathbf{q}) = k_0\,\frac{n_0}{n_c}\,\frac{|\langle \delta n\rangle|}{n_0}\sum_{\mathbf{q}'}\gamma(\mathbf{k},z|\mathbf{q}'\mathbf{q})\hat{b}(\mathbf{k},z|\mathbf{q}), \tag{28}$$

$$\gamma(\mathbf{k},z|\mathbf{q}'\mathbf{q}) = (\hat{\mathbf{e}}_{\mathbf{q}}\cdot\hat{\mathbf{e}}_{\mathbf{q}'})^2\int \frac{d\mathbf{p}}{(2\pi)^2}\,\hat{b}(\mathbf{k}-\mathbf{p},z|\mathbf{q}')G(\hat{\mathbf{p}}\cdot\mathbf{M}). \tag{29}$$

In particular, if the polarizations are orthogonal, then the interbeam transfer rate vanishes. This could be achieved, in the NIF context, if every beam in the inner cone crossed with, at most, one beam in the outer cone at some spatial location, and the polarizations were appropriately chosen.

B. Frequency shifts

If each beams has a small frequency shift $\omega_{\mathbf{q}}$ with respect to ω_0,

$$\epsilon(\mathbf{x}) = \sum_{\mathbf{q}} \epsilon(\mathbf{x}|\mathbf{q}) \exp(-i\omega_{\mathbf{q}} t), \tag{30}$$

then because the time-dependent hydro-Green's function is obtained from Eq. (7) by the replacement $\hat{\mathbf{k}}\cdot\mathbf{M} \to \hat{\mathbf{k}}\cdot\mathbf{M} - \omega/(kc_s)$, it can be shown that the transfer rate is modified from that given in Eqs. (15) and (16) to

$$\frac{\partial}{\partial z} \hat{b}(\mathbf{k},z|\mathbf{q}) = k_0 \frac{n_0}{n_c} \frac{|\langle \delta n \rangle|}{n_0} \sum_{\mathbf{q}'} \gamma(\mathbf{k},z|\mathbf{q}'\mathbf{q}) \hat{b}(\mathbf{k},z|\mathbf{q}), \tag{31}$$

$$\gamma(\mathbf{k},z|\mathbf{q}'\mathbf{q}) = \int \frac{d\mathbf{p}}{(2\pi)^2} \hat{b}(\mathbf{k}-\mathbf{p},z|\mathbf{q}') \times G[\hat{\mathbf{p}}\cdot\mathbf{M} - (\omega_{\mathbf{q}} - \omega_{\mathbf{q}'})/(pc_s)]. \tag{32}$$

If a pair of beams is very well separated, then, as in Eq. (20),

$$\gamma(\mathbf{k},z|\mathbf{q}'\mathbf{q}) \approx G[\hat{\mathbf{p}}\cdot\mathbf{M} - (\omega_{\mathbf{q}} - \omega_{\mathbf{q}'})/(pc_s)]_{\mathbf{p}=\mathbf{q}-\mathbf{q}'}, \tag{33}$$

so that the combined effect of flow and frequency shift may be interpreted in terms of an effective frequency shift and no flow, or flow and no frequency shift, *for interbeam transfer alone* between a given pair of beams. Since self-focusing is affected by flow (e.g., spatial incoherence is increased due to the combination of self-focusing and flow—see Figs. 7(a), 12, and 13 of Ref. 5, also, flow induces beam deflection), in general, the case of flow with no frequency shift is not equivalent to some case with frequency shifts alone.

If the sign and magnitude of the frequency difference is chosen appropriately, the magnitude of G, and hence transfer rate, may be reduced. Since, however, plasma flow may be inhomogeneous, power transfer could be reduced in one spatial region, but not in another.

C. Plasma inhomogeneity

Since spatial Fourier transforms are only needed in the x-y plane, variation of the plasma flow in the laser propagation direction may be accounted for by considering $\mathbf{M}$ in Eq. (7) to be a given function of z, and this change simply works its way through the formalism. If the flow changes direction so as to change the sign of the power transfer between a given pair of beams, then this weakens the magnitude of the overall transfer. However, at least for the example considered in Sec. V, in which significant transfer occurs over a distance of about 100 μm, the flow would need to vary by order unity over this relatively short distance to have much of an effect on the transfer.

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[9] The various beams need not have the same value of F, accordingly the size of the Fourier disk for each beam may be different.
[10] It is only necessary that the angle subtended by the subset of beams that actually cross in a given spatial region not be too large, and, of course, that the individual beam f/#'s be large.
[11] Perhaps the most serious limitation of the steady-state assumption is that it may be destabilized by flow along the z direction near the self-focusing threshold. See A. Schmitt, Bull. Am. Phys. Soc. **40**, 1824 (1995).
[12] See Eq. (16d) in D. F. DuBois, D. R. Nicholson, and H. A. Rose, Phys. Fluids **28**, 202 (1985).
[13] See Eq. (38) of Ref. 5.
[14] Actually, this is a generalization of the RPP result obtained in Ref. 7 because the beam spectrum is allowed to evolve.
[15] This remark discounts the indirect effect of power transfer on beam deflection that follows from the increase of a single beam's deflection rate with its power.
[16] This would be the case if the two beams and the hohlraum axis were in a plane and there was no azimuthal flow. If the radial flow is positive, then the inner cone gains power from the outer cone.
[17] The somewhat arbitrary definition of a beam's power is to assign a mode's power to, e.g., beam one, if its Fourier space location is closer to the initial location of beam one's centroid.
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[20] If self-focusing could be ignored, then the beam deflection rate would increase linearly with the average laser intensity $\sim\langle P_{\rm hs}\rangle/P_c$.
[21] See the discussion of Fig. 16 in Ref. 5.