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A similarity solution describing the collision of two planar premixed flames

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Abstract
The problem of the head-on collision of two planar premixed flame fronts is considered using the idealized model of a single step reaction controlled by the deficient species and an Arrhenius reaction law. Density changes are neglected. It is shown that a similarity solution exists if the deficient species and temperature are equidiffusive (Lewis number unity). The similarity solution, derived using activation energy asymptotics, is valid in the intermediate region when the flames are close enough that their pre-heat zones overlap but their reaction zones may be considered to be well separated. A one-dimensional numerical simulation shows good agreement with the analytical solution.

1. Introduction

The problem of the annihilation through collision of a pair of planar flames propagating in a slab of premixed gases was studied by Chen and Sohrab [6] through numerical simulation. They considered a mixture of methane and oxygen with an inert component (nitrogen) and used the four step reduced chemistry model of Seshadri and Peters. It was found that the interaction proceeded in two phases. As the flames approached close enough that their pre-heat zones overlapped, they accelerated. This was followed by a second phase when the reaction zones merged and there was a very short period of extremely large acceleration before final annihilation. They also observed that when the Lewis number of the deficient species was less than unity, the flames first slowed down due to depletion of the fresh mixture before undergoing acceleration followed by rapid acceleration and final annihilation. Subsequently, the problem was studied numerically by Echekki *et al* [9] using a more elaborate detailed chemistry model for the methane air flame. They found similar results to that of Chen and Sohrab, reporting a sevenfold increase of flame propagation speeds over that of isolated flames in the ‘radical interaction’ stage just before the mutual annihilation. Similar computational results were presented by Wichman and Vance [8] on a simplified model together with some rationalizations on possible effects of the Lewis number.

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In this paper, an analytical solution is presented for the problem of colliding premixed flames for unity Lewis numbers. The solution exhibits the qualitative behaviour observed in the earlier studies cited above. Further, numerical solution of the basic equations confirms the accuracy of the asymptotic approach based on large activation energy in a useful parameter range.

2. Formulation

We consider two identical planar premixed flames that are initially separated by a distance that is very large compared to the flame thickness. They then propagate towards each other until they come together and annihilate due to exhaustion of reactants.

For simplicity, we restrict ourselves to the simplest nontrivial situation. Thus, we assume a global irreversible one-step Arrhenius reaction and unity reaction order for the limiting species. We further assume that the heat released by chemical reactions is small compared with the thermal energy of the mixture, so that we can adopt the constant density approximation. The material constants, namely, the specific heat c_p, density ρ, thermal diffusivity D_T and mass diffusivity of the deficient species D, are assumed to be uniform throughout the mixture. We adopt a coordinate system with origin at the extinction point, and the x-axis perpendicular to the flame surfaces, as shown in figure 1. Because of symmetry, it suffices to consider only the half plane $0 \leqslant x < +\infty$. The non-dimensional conservation equations for temperature (θ) and mass fraction (y) of the limiting species are then given by

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + \mathcal{D} y \, \mathrm{e}^{-(1-\theta)/\varepsilon}, \tag{1}$$

$$\frac{\partial y}{\partial t} = \frac{1}{Le} \frac{\partial^2 y}{\partial x^2} - \mathcal{D} y \, \mathrm{e}^{-(1-\theta)/\varepsilon}. \tag{2}$$

Here, ε is the inverse of the Zel'dovich number, and Le is the Lewis number of the limiting species. The Damköhler number is taken as $\mathcal{D} = \frac{1}{2}\varepsilon^{-2}$. Therefore, in the limit of small ε, the speed of propagation of a free flame at $Le = 1$ is unity at lowest order in ε.

In the AEA limit $\varepsilon \to 0$, equations (1) and (2) reduce to

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2}, \tag{3}$$

$$\frac{\partial y}{\partial t} = \frac{1}{Le} \frac{\partial^2 y}{\partial x^2}, \tag{4}$$

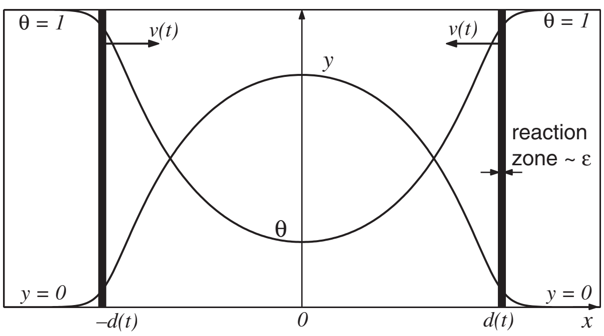

Figure 1. Profiles of temperature and mass fraction of the deficient reactant for two planar colliding premixed flames.

except in a thin ($\sim\varepsilon$) reaction zone in the immediate vicinity of $x = \pm d(t)$, which we will take as defining the location of the flame. Due to symmetry, we have the following boundary conditions at $x = 0$:

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial y}{\partial x}\right|_{x=0} = 0. \tag{5}$$

Since the limiting component is fully depleted behind the flame, we also have

$$\theta(x = d(t)) = 1, \qquad y(x = d(t)) = 0. \tag{6}$$

It only remains to write down the flux matching conditions on the unburnt side of the flame. These are obtained by asymptotic matching with the inner solution (reaction zone) and the method of deriving them is well known [3]:

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=d(t)} = \sqrt{Le}, \qquad \left.\frac{\partial y}{\partial x}\right|_{x=d(t)} = -Le^{3/2}. \tag{7}$$

Equations (3) and (4) must be solved together with boundary conditions (5)–(7). The function $d(t)$ is unknown and must be determined as part of the solution.

3. A similarity solution for unity Lewis number

Let us introduce a coordinate transformation $\eta = x/d(t)$, so that the flame position always corresponds to $\eta = \pm 1$. In terms of the new coordinates, equations (3) and (4) become

$$\frac{\partial\theta}{\partial t} - \frac{\dot{d}}{d}\eta\frac{\partial\theta}{\partial\eta} = \frac{1}{d^2}\frac{\partial^2\theta}{\partial\eta^2}, \tag{8}$$

$$\frac{\partial y}{\partial t} - \frac{\dot{d}}{d}\eta\frac{\partial y}{\partial\eta} = \frac{1}{Le}\frac{1}{d^2}\frac{\partial^2 y}{\partial\eta^2} \tag{9}$$

(the dot denotes time derivative) and the boundary conditions (5)–(7) become

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = 0, \qquad \left.\frac{\partial y}{\partial\eta}\right|_{\eta=0} = 0, \tag{10}$$

$$\theta(\eta = 1) = 1, \qquad y(\eta = 1) = 0, \tag{11}$$

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} = d\sqrt{Le}, \qquad \left.\frac{\partial y}{\partial\eta}\right|_{\eta=1} = -dLe^{3/2}. \tag{12}$$

We will look for a similarity solution. Let us define two similarity variables

$$\Theta = \frac{\theta - \theta_0(t)}{1 - \theta_0(t)}, \qquad Y = \frac{y}{y_0(t)}, \tag{13}$$

where $\theta_0(t) = \theta(t, \eta = 0)$, $y_0(t) = y(t, \eta = 0)$. If there is a similarity solution for equations (8) and (9), Θ and Y should be invariant in time so that equations (8) and (9) can be rewritten in terms of Θ and Y as a pair of ordinary differential equations (the prime denotes derivative with respect to η):

$$\Theta'' - \lambda\eta\Theta' - \mu(1 - \Theta) = 0, \tag{14}$$

$$\frac{1}{Le}Y'' - \lambda\eta Y' + \nu Y = 0, \tag{15}$$

where

$$\mu = \frac{d^2\dot{\theta}_0}{1 - \theta_0}, \qquad \nu = -\frac{d^2\dot{y}_0}{y_0}, \qquad \lambda = -d\dot{d} \tag{16}$$

are three unknown constants. The boundary conditions for Θ and Y are

$$\Theta(0) = 0, \qquad Y(0) = 1, \tag{17}$$

$$\Theta(1) = 1, \qquad Y(1) = 0, \tag{18}$$

$$\Theta'(0) = 0, \qquad Y'(0) = 0, \tag{19}$$

$$\Theta'(1) = \sqrt{Le}\frac{d}{1-\theta_0}, \qquad Y'(1) = -Le^{3/2}\frac{d}{y_0}. \tag{20}$$

Equations (16) can be solved:

$$1 - \theta_0 = C_1 d^{\mu/\lambda}, \qquad y_0 = C_2 d^{\nu/\lambda}, \tag{21}$$

where C_1 and C_2 are constants. If we use (21) to evaluate the right-hand sides of the boundary conditions (20), we find that the right-hand sides can be made time independent only if $\mu = \nu = \lambda$. Now, the solution to equations (14) and (15) may be written down in terms of the one unknown parameter λ:

$$\Theta(\eta) = 1 - \exp\left(\frac{\lambda\eta^2}{2}\right) + \lambda\eta\int_0^\eta \exp\left(\frac{\lambda\zeta^2}{2}\right)\,\mathrm{d}\zeta, \tag{22}$$

$$Y(\eta) = 1 - \Theta(\sqrt{Le}\,\eta). \tag{23}$$

However, equation (23) cannot be made consistent with the boundary condition (18) unless $Le = 1$. This means that the similarity solution exists only for the unity Lewis number case. We, therefore, assume $Le = 1$. The boundary condition (18), then provides the following transcendental equation for determining λ:

$$\frac{1}{\lambda} = \int_0^1 \exp\left[\frac{\lambda}{2}(\zeta^2 - 1)\right]\mathrm{d}\zeta, \tag{24}$$

which can be solved numerically ($\lambda = 1.710\ldots$).

If we choose the instant of annihilation of the flames as the origin of time (therefore, $t < 0$ and the flames are infinitely separated at $t = -\infty$), equation (16) is readily solved to give the flame position, $d(t)$, as a function of time:

$$d(t) = \sqrt{-2\lambda t} \simeq \sqrt{-3.42t}. \tag{25}$$

The flame velocity is then

$$v(t) = -\dot{d} = \frac{\lambda}{\sqrt{-2\lambda t}} \simeq \frac{1.71}{\sqrt{-3.42t}}, \tag{26}$$

which shows that the flames speed up on approach and the velocity becomes singular at the instant of annihilation. Comparing equations (20)–(22), we get the constants $C_1 = C_2 = \mathrm{e}^{-\lambda/2}$, so that $1-\theta_0(t) = \mathrm{e}^{-\lambda/2}d(t)$. Now, since physical solutions must satisfy $1-\theta_0(t) \leqslant 1$, we have

$$d(t) \leqslant d_{\mathrm{cr}} = \mathrm{e}^{\lambda/2} \simeq 2.35, \tag{27}$$

$$t \geqslant t_{\mathrm{cr}} = -\frac{\mathrm{e}^\lambda}{2\lambda} \simeq -1.62. \tag{28}$$

This gives a constraint for d and t for the similarity solution to exist. The critical values, d_{cr} and t_{cr}, may be regarded as the relevant length and time scale that separate 'the flame interaction stage' when the pre-heat zones of the two fronts overlap from the 'free flame stage' during which there is essentially no interaction.

4. Comparison with numerical solution

Equations (1) and (2) were numerically integrated for $Le = 1$, and the results were compared with the AEA solutions presented in the last section. In the simulation, we used a sixth-order compact finite difference scheme [5] for the discretization of spatial derivatives. A fourth-order Runge–Kutta time stepping was employed and the time step was properly chosen to satisfy the requirements of stability and accuracy.

In the last section, we presented a similarity solution for the flame interaction stage. So, it would seem prudent to examine the numerical solution for evidence of self-similar collapse. Figure 2 shows the similarity variable Θ as a function of η at several time instants during the collision process. In the numerical simulations, the reaction zone is of finite, though small thickness, so there is a slight ambiguity as to the meaning of 'flame position'. We identify the flame position with the location of the maximum reaction rate. As can be seen, the similarity variable Θ constructed from the numerical solution exhibits good self-similar collapse when d is in the range $0.65 < d < 1.87$, and the profile is in close agreement with the asymptotic solution for Θ. However, when $d < 0.65$ the distance between the flames is comparable to the reaction zone thickness. Thus, the reaction zones of the two colliding flames begin to merge, so the asymptotic analysis breaks down and the self-similarity is also lost.

Figure 3 compares numerical and asymptotic results for the evolution of flame propagation velocity with time for three Zel'dovich numbers. It is clear that the agreement between asymptotics and numerical calculations becomes better with increasing value of Zel'dovich number as expected. Figure 4 presents the same data as the ratio of the computed flame speed to the theoretical speed (26) as a function of the distance d. It is clear that for $d < d_{\mathrm{cr}} = 2.35$, good collapse of the numerical data on the theoretical curve is obtained with better agreement for smaller values of ε. However, the agreement once again starts getting worse as $d \to \varepsilon$

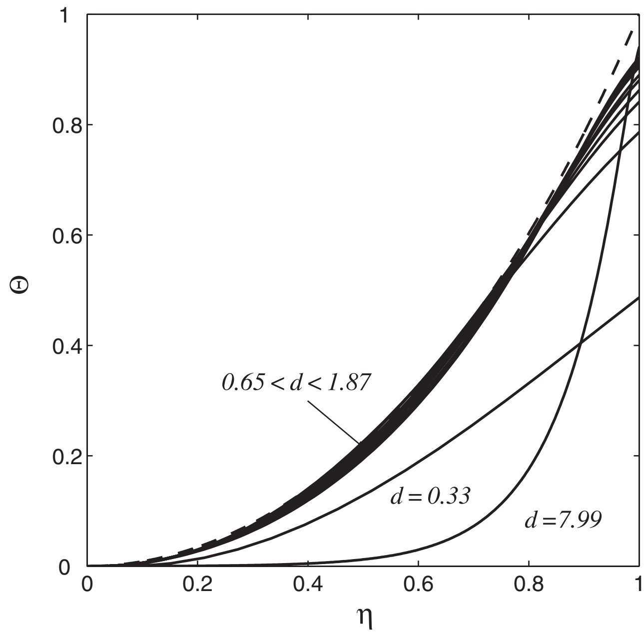

Figure 2. Temperature profiles in self-similar variables. ——: numerical solutions; - - - -: AEA solution. Parameters are: $\varepsilon^{-1} = 16$, $Le = 1$.

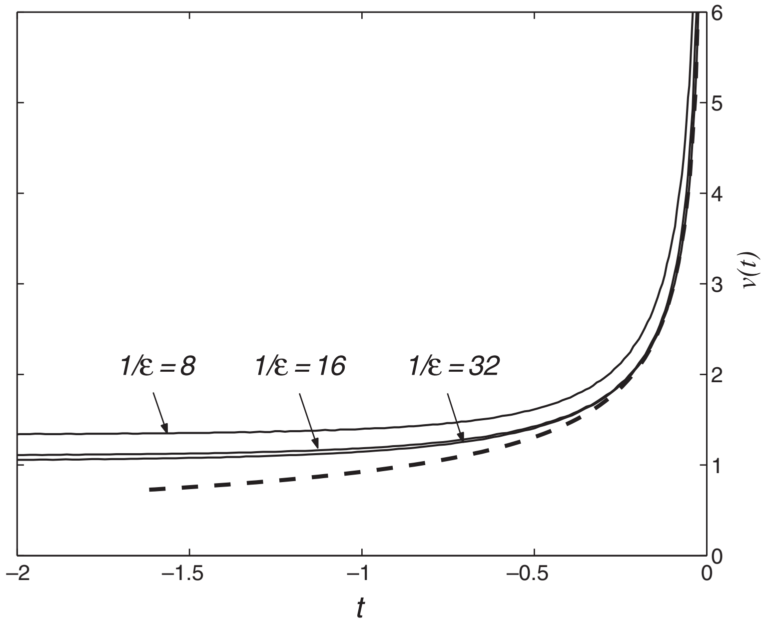

Figure 3. Evolution of flame propagation velocity for various Zel'dovich numbers and a fixed (unity) Lewis number. ——: numerical solutions; - - - -: AEA solution.

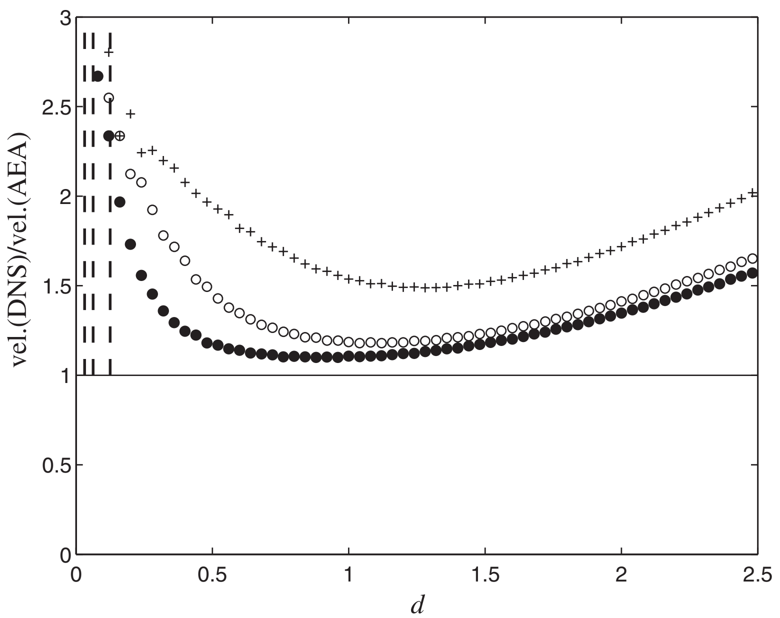

Figure 4. The ratio of the flame speed from direct numerical simulation (DNS) to that predicted by AEA (equation (26)) as a function of half the distance between the flames, $d(t)$ for $1/\varepsilon = 8$ (+), 16 (○) and 32 (●). Dashed vertical lines indicate the locations $d = \varepsilon$, a measure of the extent of the reaction zone.

since now the reaction zones start to overlap, invalidating the AEA limit assumption that the reaction zone is infinitely thin.

5. Conclusion

The problem of the collisional annihilation of a pair of planar premixed flames was considered in a medium where the burning rate is controlled by a single deficient species. A single step reaction model with Arrhenius chemistry was assumed. The problem was analysed in the large activation energy limit, where the pre-heat zones are resolved, but the reaction zones are replaced by jump conditions on the fluxes of temperature and species. Density variation was neglected.

A similarity solution was found for the temperature and species concentration. The solution is valid provided that half the distance between the two flames is less than $d_{\mathrm{cr}} = 2.35$ flame units. The propagation speed of the flames becomes singular at the moment of collision of the reaction fronts. The velocity is inversely proportional to the square root of the time remaining to the moment of collision of the reaction fronts. The singularity is a consequence of the infinite Zel'dovich number limit that collapses the reaction zones to infinitely thin reaction fronts within which the reaction rates are infinitely large. The similarity solution is valid only in the case of unity Lewis number.

The solution to the reduced problem as described above was computed numerically for large but finite Zel'dovich numbers (ε^{-1}). Thus, the numerical solution adopts all the simplifying assumptions above, except for the one implied by AEA ($\varepsilon \to 0$). The numerical solution shows close agreement with the similarity solution presented except

(i) when the reaction zones of the two flames overlap;
(ii) when the flames are further apart than $d_{\mathrm{cr}} = 2.35$ flame units, in which case, the self-similar solution is unphysical.

The case of non-unity Lewis numbers results in some interesting new qualitative effects that can be addressed by a natural extension of the approach presented here. This generalization will be addressed in a future publication. The constant density approximation is valid only when the reactants are greatly diluted by an inert species. In general, thermal density fluctuations could have a significant qualitative effect on the problem. Since the thermal expansion of the gas would want to 'blow apart' the two flames, the time to annihilation is expected to be longer in the presence of density variations. Further, if perturbations from the planar configuration are allowed, complex effects due to the interaction of differential diffusion, thermal expansion and geometry are also possible.

In the 'flamelet' [1] picture for turbulent premixed combustion, a new flame surface is continuously created by flame surface wrinkling by the turbulent fluid and eliminated because of strain induced extinction and mutual annihilation. The analysis presented here provides a fundamental understanding, albeit within the context of an idealized model, of the latter process.

In turbulent non-premixed combustion the flame is often pictured as a highly convoluted 'flame sheet' that may be locally modelled as a simple counterflow diffusion flame [10]. Strain rate fluctuations can lead to 'tears' in the flame sheet with a characteristic 'edge flame' structure bridging the extinct and burning zones [11]. Provided that the strain rate is not very high, the edge flame takes the form of a 'triple-flame' when viewed in cross-section [7]. In the limit of very small strain rates, the premixed branch curvature of a triple flame is very small and in this case the problem studied in this paper should provide an approximate description of the collision of a pair of triple flames (at unit Lewis numbers). It is of interest to note that the problem of the collision of edge flames has recently been studied [2] using the 'one-dimensional approximation' of Buckmaster [4], and equation (25) is recovered in the limit $t \to 0$ even though the physical system is a diffusion flame instead of the planar premixed flame considered in this paper.

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