

LETTERS

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Effect of induced spatial incoherence on flow induced laser beam deflection: Analytic theory

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Analytic results are presented for the laser beam deflection rate in the case of spatial and temporal smoothing by induced spatial incoherence (ISI). It is shown that for flow perpendicular to the beam propagation direction with Mach number M , temporal smoothing decreases the beam deflection rate for $M > 1$, but may increase it for $M < 1$ and weak acoustic wave damping. © 1997 American Institute of Physics. [S1070-664X(97)01712-6]

The ability to accurately control the pointing of high power laser beams is critical to achieving spherically symmetric implosions of capsules in inertial confinement fusion. Indirect drive hohlraums typically have cylindrical symmetry and, owing to the large number of laser beams that will be employed at the National Ignition Facility (NIF),¹ the laser irradiance, nearly will as well. Nearly spherical, indirect, illumination of the capsule may be achieved by precisely aiming the laser beams into rings whose position along the hohlraum axis is chosen to zero out low order azimuthal asymmetry, while high order asymmetries are highly attenuated by a sufficiently large hohlraum-to-capsule aspect ratio. To engineer this precision aiming, laser beam trajectories through the hohlraum plasma must be accurately predicted. At present, the main design tool, LASNEX,² only allows for the classical effect of refraction by macroscopic plasma density non-uniformity, while experimental observation³⁻⁷ of nonclassical beam deflection has been attributed to the non-linear combined effect of flow transverse to the laser beam⁸⁻¹¹ and microscopic density fluctuations induced by laser speckles or "hot spots." Some previous theoretical work^{8,10} of beam deflection mainly presented numerical results of models which also incorporated self-focusing, while the relative simplicity of a model which takes the laser intensity pattern to be time independent and identical to that produced by random phase plate (RPP) optics¹² in a quiescent plasma has allowed us to obtain⁹ detailed analytic predictions of the beam deflection rate. In relatively weak regimes of laser intensity, such that most hot spots¹³ are below the power threshold for explosive self focusing,¹⁴ such a model is plausible, as well as in stronger intensity regimes where self-focusing is suppressed by strongly supersonic flow.^{8,15} In such regimes beam deflection may yet be significant, since it has no intensity threshold. The validity of our

model, which ignores self focusing, may be extended by applying temporal smoothing sufficient to suppress hot spot self-focusing, while keeping the average laser intensity small enough to remain below the linear, collective filamentation threshold.¹⁶ In this paper, an analytic theory of beam deflection by flow is presented for the case of a laser beam which is spatially smoothed by a RPP and temporally smoothed by induced spatial incoherence (ISI).¹⁷

The standard model of temporal beam smoothing by ISI is to assign statistically independent, random, time varying phases to each Fourier mode of the laser light.¹⁷ The electric field at the focal plane ($z=0$) may therefore be modeled as

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \Re[\epsilon(\mathbf{x}, t) \exp(i\omega_0 t)], \quad (1)$$

where

$$\epsilon(\mathbf{x}, t) = \sum_{|\mathbf{k}| < k_m} \exp[i\{\mathbf{k} \cdot \mathbf{x} + \phi(\mathbf{k}, t)\}]. \quad (2)$$

Here $\mathbf{k} = (k_x, k_y)$, $\mathbf{x} = (x, y)$, and, $k_m = k_0 / (2F) = \pi / (F\lambda)$, where $k_0 = 2\pi / \lambda$, F is the optic "F" number, λ and ω_0 are the wavelength and angular frequency of the laser. The phase lag introduced by each echelon element, $\phi(\mathbf{k}, t)$, is an independent random variable with an auto-correlation time of the order of $1/\Delta\nu$, where $\Delta\nu$ is the frequency bandwidth. The amplitude E_0 varies slowly in \mathbf{x} and defines the diffraction limited intensity profile of a single RPP element.

The beam deflection rate can be related to the self-correlation of the ponderomotive potential $U(\mathbf{x}, t) \propto |\epsilon(\mathbf{x}, t)|^2$ at two space time points, $C(\mathbf{R}, \tau) = \langle U(\mathbf{x}, t) U(\mathbf{y}, t') \rangle / \langle U^2(\mathbf{x}, t) \rangle$, where $\mathbf{R} = \mathbf{x} - \mathbf{y}$, $\tau = |t - t'|$

and $\langle \rangle$ denotes ensemble average. For the laser light model (1), we have, when $N \gg 1$ (N is the total number of beamlets into which the beam is split)

$$C(R, \tau) = \frac{1}{2}[1 + f(\tau)C_0(R)], \quad (3)$$

where

$$C_0(R) = \frac{4}{R^2 k_m^2} J_1^2(Rk_m) \quad (4)$$

[$J_1(x)$ is the Bessel function of order unity], and

$$f(\tau) = |\langle \exp[i\{\phi(\mathbf{k}, t) - \phi(\mathbf{k}, t')\}] \rangle|^2. \quad (5)$$

The function $f(\tau)$ is non-negative, it has a width of the order of $1/\Delta\nu$, and $f(0) = 1$. The precise form of $f(\tau)$ will depend

on the details of the method of creating the bandwidth, but, for the purpose of analytical work we will assume a simple form

$$f(\tau) = \frac{T^2}{T^2 + \tau^2}, \quad (6)$$

where $T \sim 1/\Delta\nu$.

The density response of the plasma due to an applied ponderomotive potential $U(\mathbf{x}, t)$ may be written in the linearized approximation⁸ as

$$\widehat{\delta n} = n_0 G(\mathbf{k}, \omega) \hat{U}(\mathbf{k}, \omega), \quad (7)$$

where n_0 is the equilibrium density, the ‘‘hat’’ denotes Fourier-transform with respect to x , y and t , and,

$$G(\mathbf{k}, \omega) = \frac{-Z/m_i c_s^2}{\{1 - [(\omega/c_s k) - M(k_x/k)]^2 - 2i\gamma_0[(\omega/c_s k) - M(k_x/k)]\}}. \quad (8)$$

In (8), M is the Mach number of the incident flow, c_s is the ion-acoustic sound speed, m_i is the ion mass, Z is the charge state and γ_0 is the nondimensional Landau damping coefficient. The rate of beam deflection may be written⁸ as

$$\frac{d\alpha}{dz} = -\frac{1}{2n_c} \frac{\int d\mathbf{x} U \partial_x n}{\int d\mathbf{x} U} \quad (9)$$

where n_c is the critical plasma density. For beam diameter large compared to the intensity fluctuation correlation length, the numerator and denominator of (9) may be replaced by the corresponding ensemble averages to obtain

$$\frac{d\alpha}{dz} = -\frac{n_0}{n_c} \langle U \rangle \int d\mathbf{k} d\omega i k_x G(\mathbf{k}, \omega) \hat{C}(\mathbf{k}, \omega), \quad (10)$$

where the Green function is given by (8) and we have used the relation $\langle U^2 \rangle = 2\langle U \rangle^2$. It may be shown from (10), that in the limit of infinitely large correlation times, the beam deflection reduces to the corresponding expression (Eq. (145) of Ref. 9) for RPP optics while for infinitely short correlation times the deflection vanishes. In the limit when $\gamma_0 \ll 1$, the ω -integration can be performed using the well-known relation

$$\int_{-\infty}^{+\infty} \frac{f(x)}{x \mp i\epsilon} \sim \epsilon \rightarrow 0 P \int_{-\infty}^{+\infty} \frac{f(x)}{x} dx \pm i\pi f(0).$$

We get, after some simplification,

$$\frac{d\alpha}{dz} = -\frac{Z\pi\langle U \rangle}{4m_i c_s} \frac{n_0}{n_c} \int_0^\infty dk k^3 \hat{C}_0(k) \int_0^{2\pi} d\theta [\hat{f}]_\pm \cos \theta, \quad (11)$$

where $[\hat{f}]_\pm \equiv \hat{f}[c_s k(M \cos \theta + 1)] - \hat{f}[c_s k(M \cos \theta - 1)]$. For the model (6) we have $\hat{f}(\omega) = (T/2)\exp(-|\omega|T)$. We now write $-k^2 \hat{C}_0(k)$ as a Fourier-inverse of $\nabla^2 C_0$ and express

the angular integrals in the Fourier-inverse in terms of the zero-order Bessel function. The integral over k is then performed using well-known properties of Bessel functions so that (11) reduces, after a fair amount of algebra, to

$$\begin{aligned} \frac{F\lambda}{\pi} \frac{d\alpha}{dz} = & -\frac{3}{4} \frac{n_0}{n_c} S g^2 \int_0^\infty dr r^2 C'_0(r) \\ & \times \int_0^\pi \frac{|1 - M \cos \theta| \cos \theta d\theta}{[r^2 + g^2(1 - M \cos \theta)^2]^{5/2}}, \end{aligned} \quad (12)$$

where we have introduced the dimensionless parameters $g = \pi c_s T / (F\lambda)$ and $S = Z\langle U \rangle / m_i c_s^2 = 1/4 (v_0/v_e)^2$, where v_0 is the electron quiver velocity based on the average laser intensity and v_e is the electron thermal speed, and, the dimensionless variable $r = Rk_m = \pi R / F\lambda$. Physically, g is the

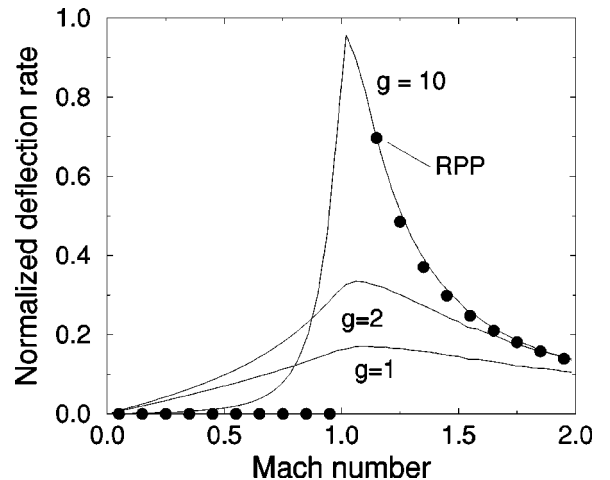


FIG. 1. Normalized (see the text) beam deflection rate as a function of Mach number for several fixed values of g together with the corresponding result for an RPP with no temporal smoothing.

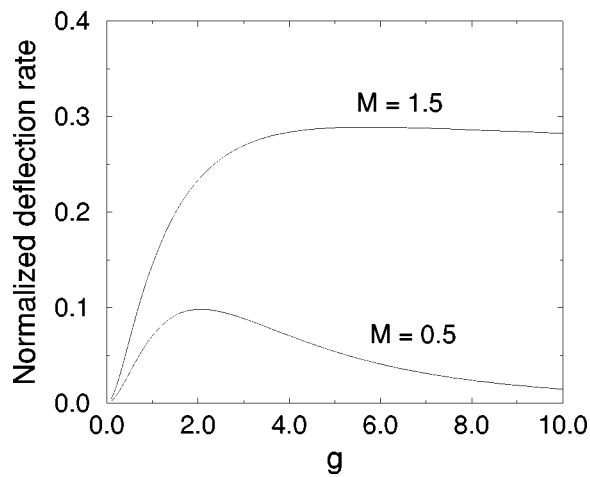


FIG. 2. Normalized beam deflection rate as a function of g for fixed Mach numbers $M=0.5$ and 1.5 .

ratio, of the distance traversed by the ion-acoustic sound wave in a correlation time, to the hot-spot diameter. For parameters typical of ISI, g is of order unity. The parameter S is a measure of the mean laser intensity. The present linearized theory requires⁹ $S \ll 1$. For NIF parameters, $S \sim 0.01$, therefore, the present linearized theory is justified when self-focusing is weak.

In the limit of large correlation times ($g \rightarrow \infty$), it may be shown, that the right hand side of (12) $\sim 1/g^3$ if $M < 1$, and, $\sim KS/M\sqrt{M^2-1}$ if $M > 1$. The latter result is also true if g is held fixed, but $\sqrt{M^2-1} \gg 1/g$. Here $K = (n_0/n_c) \times (F\lambda/\pi\ell)$ is a dimensionless parameter, and, $\ell \approx 0.70F\lambda$ is a correlation length defined by $1/\ell = -\int_0^\infty \{C'_0(R)/2R\} dR$. This is identical to known results⁹ for the beam deflection rate due to an RPP in the absence of Landau damping or temporal smoothing. In the limit of small correlation times ($g \rightarrow 0$), it follows from (12), that, the beam deflection rate vanishes as $\sim g^2$ (the double integral in (12) can be shown to be finite as $g \rightarrow 0$). These scalings are expected to hold if (6) is replaced by any $f(\tau)$ with the same long and short time behavior which control the small and large g limits, respectively.

Figure 1 shows the normalized beam deflection rate, $(F\lambda/\pi)(d\alpha/dz)$ in units of $S(n_0/n_c)$, obtained by numerically integrating (12), for several values of g . Two qualitative features should be noted. First, the beam deflection rate for $M > 1$ is reduced by temporal smoothing. This was to be expected since temporal smoothing causes a partial decorrelation between the ponderomotive force and the density response. The second, and perhaps an unanticipated feature is, for $M < 1$, temporal smoothing results in an increased deflection rate. The reason for this is, with a finite g , a sound wave may be resonantly driven (so that the denominator in the Green function (8) vanishes), even if $M < 1$. In the absence of temporal smoothing, a resonant denominator was possible only for $M > 1$. The integrated deflection can therefore be either increased or decreased by temporal smoothing depending on the details of the plasma flow in the hohlraum. These features are more clearly seen in Fig. 2 which shows the variation of the beam deflection rate with g for two fixed values of the Mach number $M=0.5$ and $M=1.5$. If a typical

hot-spots power exceeds the threshold for self-focussing then the increased deflection due to the resonant denominator may be more than compensated by the reduction due to suppression of self-focussing. Our conclusion that temporal smoothing actually increases the deflection-rate for $M < 1$ may not be true under those circumstances. This conclusion may also be false in the presence of Landau damping, which may be the dominant effect if g is large and $M < 1$. We may get an estimate for the upper bound of g by observing that the subsonic deflection $\sim 1/g^3$ when g is large ($\gamma_0=0$) while the corresponding deflection due to Landau damping alone $\sim \gamma_0$ ($g=\infty$) provided⁹ $1-M \gg \sqrt{\gamma_0}$. For $M > 1$ the deflection is dominated by the ‘‘wave-drag’’ effect⁹ and a small Landau damping is not significant. Thus Landau damping may be neglected when $g \ll \gamma_0^{-1/3}$. Near $M=1$, nonlinear effects (neglected here), finite g effects and Landau damping all compete to regularize the singularity in the beam deflection rate. However, since this singularity is integrable, the behavior very close to $M=1$ has only a small effect⁹ on the physically relevant integrated deflection.

The above analysis was presented for the ISI method of temporal smoothing. However, similar results appear to hold for the random phase modulation¹⁸ variant of ‘‘Smoothing by Spectral Dispersion’’ (SSD), in the limit of a large number of color cycles.

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- ¹J. Lindl, Phys. Plasmas **2**, 3933 (1995).
- ²G. Zimmerman and W. Kruer, Comments Plasma Phys. Control. Fusion **2**, 51 (1975).
- ³J. Moody, B. MacGowan, D. Hinkel, W. Kruer, E. Williams, K. Estabrook, T. Shepard, R. Kirkwood, and R. Berger, Phys. Rev. Lett. **77**, 1294 (1996).
- ⁴S. G. Glindinning, L. Powers, R. L. Kauffman, O. L. Landon, D. B. Ress, G. F. Stone, L. J. Suter, and A. L. Richard, private communication (1996).
- ⁵B. Bauer, private communication (1996).
- ⁶N. Delamater, T. Murphy, A. Hauer, R. Kauffman, A. Richard, E. Lindman, G. Magelssen, B. Wilde, D. Harris, B. Failor, J. Wallace, L. Powers, S. Pollaine, L. Suter, R. Chrien, T. Shepard, H. Rose, E. Williams, M. Nelson, M. Cable, J. Moore, M. Salazar, and K. Gifford, Phys. Plasmas **3**, 2022 (1996).
- ⁷N. Delamater, T. Murphy, A. Hauer, R. Kauffman, A. Richard, E. Lindman, G. Magelssen, B. Wilde, L. Powers, S. Pollaine, L. Suter, R. Chrien, D. Harris, M. Cable, J. Moore, K. Gifford, and R. Wallace, in *Laser Interaction and Related Plasma Phenomenon, Proceedings of the 12th International Conference, Osaka, Japan* (American Institute of Physics, Woodbury, NY, 1995).
- ⁸H. Rose, Phys. Plasmas **3**, 1709 (1996).
- ⁹S. Ghosal and H. Rose, Phys. Plasmas **4**, 2376 (1997).
- ¹⁰D. Hinkel, E. Williams, and C. Still, Phys. Rev. Lett. **77**, 1298 (1996).
- ¹¹W. Kruer and J. Hammer, Comments Plasma Phys. Control. Fusion **18**, 85 (1997).
- ¹²Y. Kato and K. Mima, Appl. Phys. B: Photophys. Laser Chem. **29**, 186 (1982).
- ¹³H. Rose, Phys. Plasmas **2**, 2216 (1995).
- ¹⁴We restrict attention to the regime where ponderomotive effects dominate thermal
- ¹⁵R. Short, R. Bingham, and E. Williams, Phys. Fluids **25**, 2302 (1982).
- ¹⁶H. Rose and D. DuBois, Phys. Fluids B **5**, 590 (1993).
- ¹⁷R. Lehmburg, A. Schmitt, and S. Bodner, J. Appl. Phys. **62**, 2680 (1987).
- ¹⁸J. Rothenberg, D. Eimerl, M. Key, and S. Weber, Proc. Soc. Photo-Opt. Instrum. Eng. **2633**, 162 (1995).