# Syllabus for ME 454: Numerical Methods in Optimal Control of Nonlinear Systems

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**URL:** Blackboard at *https://courses.northwestern.edu/webapps/login/*
**Textbook:** The required textbooks are *Functional Analysis and Linear Control Theory (Dover Books on Engineering)* by J. R. Leigh (available on amazon.com for about $10) and *Iterative Methods for Optimization* by C.T. Kelley (available online).
**Software:** *Mathematica* (available in computer labs as well as a student edition that is quite inexpensive)
**Topics Covered:** This course will cover methods in numerical optimization and optimal control with an emphasis on engineering applications and computation. Topics include differentiation, gradient descent, Newton's method, optimal control, and optimal switching control. Examples will be drawn largely from aerospace, robotics, and biomedical applications. Students will be expected to complete numerical optimization exercises in homework sets as well as a final project.
**Prerequisites:** The prerequisites are differential equations and systems analysis. Undergraduate controls and/or signal processing course would satisfy this requirement. A graduate-level systems course is also helpful, but not necessary.

The class outline is roughly as follows. Examples of applying these numerical methods in applications will be used throughout the homework sets and will focus on robotics, aviation, and locomotion.

1. Types of problems we care about

   (a) vehicle control
   (b) motion planning
   (c) motor function in humans

2. Differentiation in finite and infinite-dimensional spaces

   (a) definitions of the derivative
   (b) finite dimensional differentiation of algebraic functions
   (c) finite dimensional differentiation of integral costs
   (d) infinite dimensional differentiation of integral costs

3. Numerical Optimization: Steepest Descent and line searches
   (a) necessary conditions for optimality
   (b) the Armijo rule and sufficient decrease
   (c) optimization using gradient descent
   (d) constrained optimization using projections

4. Numerical Optimization: Newton's method and trust region methods

   (a) sufficient conditions for optimality
   (b) trust region methods
   (c) optimization using Newton's method
   (d) constrained optimization using projections and derivatives of projections

5. Finite dimensional systems with algebraic costs

   (a) simple examples of numerical optimization using gradient descent and Newton's method in both the unconstrained and constrained cases
   (b) discrete mechanics and optimal control (DMOC) using projection-based optimization

6. Finite dimensional systems with integral costs (Hybrid Systems)

   (a) examples of hybrid systems
   (b) optimization with respect to initial conditions
   (c) switching time optimization
   (d) hybrid estimation using switching time optimization
   (e) airplane mode detection and haptics mode detection

7. Infinite dimensional systems: Linear Systems and LQR

   (a) direct calculation of the LQR controllers
   (b) Riccati equations and linear optimal control
   (c) the maximum principle and the Hamilton Jacobi-Bellman equations (no proofs will be discussed on this topic)

8. Infinite dimensional systems: Nonlinear Systems

   (a) the maximum principle applied to nonlinear systems
   (b) Gradient descent for nonlinear systems using LQR controllers
   (c) Newton's method for nonlinear systems using LQR controllers
   (d) the maximum principle applied to hybrid, nonlinear systems

**Testing, Homework, Grading**

Grading will nominally be based on the following.

Homework: 50% of total grade.
Participation: 25% of total grade
Final Project: 25% of total grade.

Note that there will be no final exam and that the final project will take its place.