Numerical Determination of Mechanical Elastic Constants of Textile Composites

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ABSTRACT

This paper presents a novel procedure for predicting the effective nonlinear elastic moduli of textile composites through a combined approach of the homogenization method and the finite element method. The homogenization method is used first to obtain the effective elastic moduli of the fiber yarn based on the properties of the constituent phases. A unit cell is then built to enclose the characteristic periodic pattern in the composites. Various numerical tests such as uni-axial tension and trellising test are performed by 3D finite element analysis on the unit cell. Characteristic behaviors of force versus displacement are obtained. Meanwhile, trial mechanical elastic constants are imposed on a four-node shell element with the same size as the unit cell to match the force-displacement curves. The effective nonlinear mechanical stiffness tensor is thus obtained numerically as functions of elemental strains. The procedure is exemplified on a plain weave glass composite and is validated by comparing with 30-degree bias trellising and bi-axial tensile test results.

INTRODUCTION

Textile composite materials have recently received considerable attention, due to their structural advantages of high specific-strength and high specific-stiffness as well improved resistance to impact. Compared with unidirectional composites, the interlacing of fiber bundles in textile composites prevents the growth of damage and hence provides an increase in impact toughness. Besides their advantageous mechanical properties, textile composites are easy to handle and have excellent formability and hence are widely employed in aircraft, boat and defense industry. [1]

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To fully understand the mechanical behavior of textile composites during processing and application in order for optimal design, it is essential to obtain the effective material properties of fabric composites from the known material properties of the constituent phases.

In recent years, many efforts have been given to the estimation of effective material properties of composite materials [1-11]. The approaches developed include the homogenization method, the finite element method, analytical model and experimental approach. Due to the immense variety of available composite materials and possible fabric construction geometry, it is impractical and very time-consuming to obtain material characterizations of various composites by an experimental approach. Analytical methods, on the other hand, cannot deal with complex fabric construction geometries. Though the homogenization method is effective in predict material properties, the huge computational cost limits its application in simulating the forming of complex structures as textile composites.

By integrating the advantages of the conventional finite element formulation and the homogenization method, this paper presents a novel procedure for predicting the effective nonlinear elastic moduli of textile composites. These nonlinear effective moduli can be incorporated in a user material subroutine associated with shell elements so that efficient and accurate FEM simulation of composite sheet forming is feasible. First, based on the properties of the constituent phases, the homogenization method is employed to predict the effective elastic constants of the fiber yarn, which is regarded as a unidirectional composite. A unit cell is then built to enclose the characteristic periodic pattern in the textile composites. Using the unit cell, various numerical tests can be performed. By correlating the force versus displacement curves of the unit cell and a four-node shell element with the same size as the unit cell, the effective nonlinear mechanical stiffness tensor can be obtained numerically as functions of elemental strains. The procedure is exemplified on a plain weave glass composite and is validated by comparing with 30-degree bias trellising and bi-axial tensile test results.

MATERIAL CHARACTERIZATION OF FIBER YARNS

A plain weave E-glass/PP composite is used in this paper to illustrate the procedure of determining the material properties of the fiber yarn. The material properties of the constituent phases are listed in Table I. The volume fraction of the E-glass is 70%. The fiber yarns of the plain weave composites are regarded as unidirectional composites. A unit cell is built for the fiber yarns. The elastic moduli are assumed to be linear and orthotropic. The homogenization method is applied on this unit cell to get the effective elastic constants of the fiber yarns. Details can be found in [12]. The predicted elastic constants for the fiber yarns are:

$$E_l = 51.92GPa, \quad E_t = 21.97GPa, \quad \nu_{lt} = 0.2489$$
$$\nu_{tt} = 0.2143, \quad G_{lt} = 8.856GPa, \quad G_{tt} = 6.250GPa$$

where l represents the longitudinal direction and t denotes the transverse direction.

TABLE I. MATERIAL PROPERTIES OF E-GLASS/PP COMPOISTE

Property	Unit	E-Glass	PP
Axial Modulus	GPa	73.1	3.45
Transverse Modulus	GPa	73.1	3.45
Axial Poisson's Ratio	_	0.22	0.35
Transverse Poisson's Ratio	_	0.22	0.35
Axial Shear Modulus	GPa	30.19	1.83
Density	Kg/m^3	2540	900

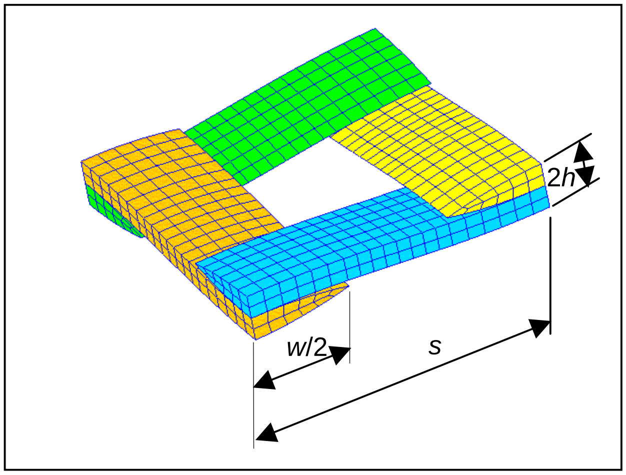

FIGURE 1. Unit cell for plain weave

UNIT CELL FOR PLAIN WEAVE COMPOSITES

The geometric description for plain weave composites presented by McBride and Chen [13] is used in this paper to model a unit cell for the plain weave composite.
The unit cell, as shown in Fig. 1, is defined by four sinusoidal curves in terms of yarn width *w*, yarn spacing *s*, and fabric thickness *h* to represent the periodic rpattern in the composite. The cross-sections of fiber yarns are approximated as circular arcs. The characteristic values of *w*, *s* and *h* are averaged from 10 measurements as

$$w = 3.44mm, \quad s = 5.28mm, \quad h = 0.40mm$$

The unit cell is discretized by 3-D 8-node continuum elements. Each fiber yarn is modeled by 304 elements. The effective elastic constants obtained from the homogenization method are imposed on the fiber yarns. The pin-jointed net idealization is assumed along the four corner lines of the unit cell. Contact conditions are prescribed between the possible interlacing surfaces of the fiber yarns under loading. Unless specified, the friction coefficient in the surface contacting is assumed to be 0.05. Boundary conditions will be modified to reflect the periodic boundary conditions of the macrostructure under different loading conditions. Numerical testes will be applied to this unit cell to obtain the effective mechanical stiffness tensor of the plain weave composite as described as follows.

NUMERICAL TESTS ON UNIT CELL

The behavior of textile composites during the shaping process is very different from that of a sheet metal. During the sheet-metal forming process, the blank is usually subjected to large extension strains. The fabric yarns, on the contrary, undergo small extension along the yarn directions while experiencing large angular variation between weft and warp yarns (a phenomenon sometimes called trellising). Experimental studies [4, 5] in textile composites showed that the yarn buckles immediately under a compression load. Hence, the fabric compressive stiffness is negligible. Consequently, the modeling used for the fabric behavior will account for large deformation and consider geometric non-linearity.

Two numerical tests will be done on the unit cell. One is trellising [14]. The other is the uni-axial extension test. The load-displacement curves are shown in Figs 2 and 3. As shown in Fig. 2, the reaction force under trellising test grows nonlinearly with the cross-corner stretch. The reaction force is very small initially, but increases sharply after a certain value of stretch, which corresponds to the shear locking angle in plain weave composites [14]. On the contrary, the reaction force in the simple extension test varies linearly with the stretch, as shown in Fig. 3. Figure 4 shows the resulting shrinkage in the direction perpendicular to the extension direction under certain stretch in the extension test.

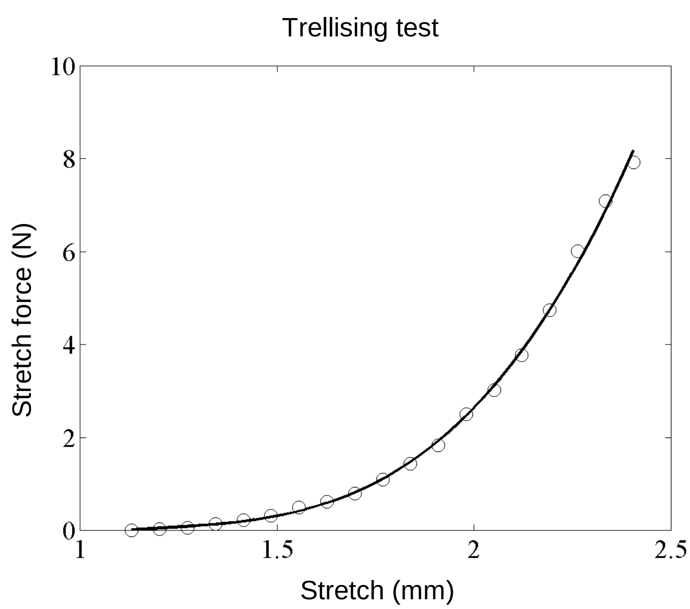

FIGURE 2. Reaction force versus stretch.

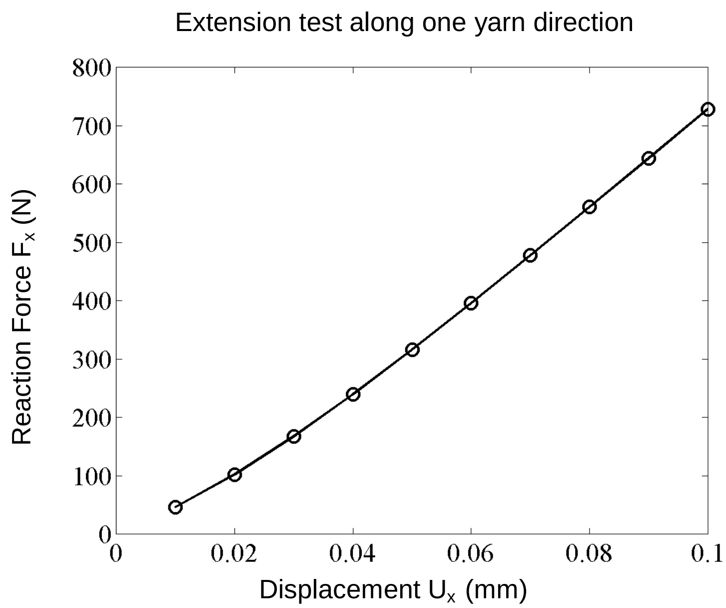

FIGURE 3. Reaction force versus stretch.

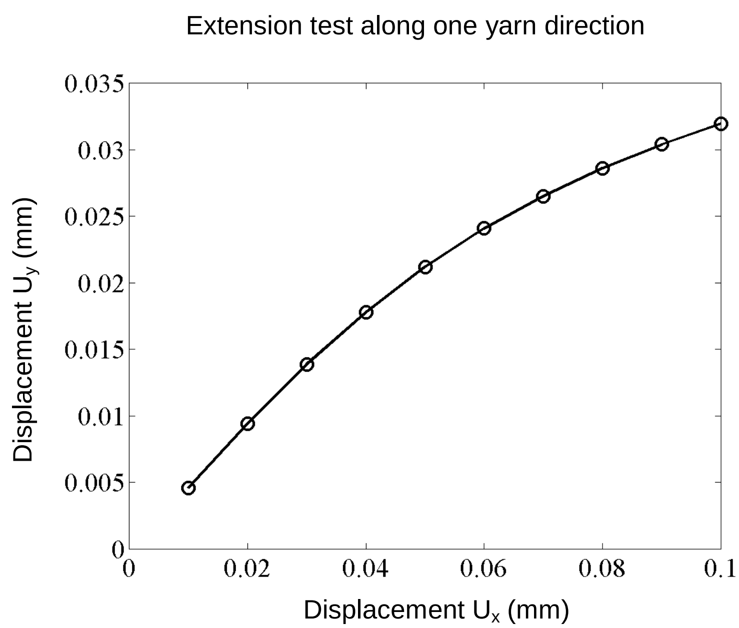

FIGURE 4. Stretch U_x versus shrinkage U_y.

EFFECTIVE MECHANICAL STIFFNESS TENSOR

A four-node shell element with the same outer size of the unit cell as in Fig. 1 is built. Large deformation and geometric non-linearity are taken into account in the

FEM analysis. Plane stress situation is taken for the shell element. The elastic moduli are assumed to be orthtropic. The material constitutive equation is given as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{1}$$

where

$$\frac{E_1}{\nu_{12}} = \frac{E_2}{\nu_{21}} \tag{2}$$

Trial mechanical stiffness tensor is modified and then imposed to the shell element in each increment of the FEM analysis to match the force-displacement curves of the unit cell under trellising and simple extension tests, respectively. Series of elastic shear modulus G_{12}, tensile modulus E_1 and Poisson's ratio ν_{12} are obtained under different stretches. Now the task is to find out the equations for describing the material elastic moduli. We propose the following relations:

$$G_{12} = f(\gamma_{12}),\ E_i = f[\sqrt{\varepsilon_i(\varepsilon_1+\varepsilon_2)}],\ \nu_{12} = f(\varepsilon_1+\varepsilon_2) \tag{3}$$

Consequently, the numerically obtained shear modulus G_{12}, tensile modulus E_1 and Poisson's ratio ν_{12} are drawn in Figs 5, 6 and 7 in circles, respectively. The solid lines in these figures denote curve-fitting values.

The curve-fitting equations for the shear modulus G_{12}, tensile modulus E_1 and Poisson's ratio ν_{12} are:

$$G_{12} = 665.66\gamma_{12}^3 - 474.84\gamma_{12}^2 + 113.55\gamma_{12} - 8.8468 \tag{4}$$

$$E_i = \begin{cases} \text{A small positive constant, e.g. 500} & \varepsilon_i < 0 \\ 5.803\times10^8 A^3 - 3.931\times10^7 A^2 + 9.079\times10^5 A + 3.98\times10^3, & \varepsilon_i > 0 \end{cases} \tag{5}$$

$$A = \sqrt{|\varepsilon_i(\varepsilon_1+\varepsilon_2)|} \tag{6}$$

and

$$\nu_{12} = \begin{cases} 0.0135, & \varepsilon_1+\varepsilon_2 > 0.021 \\ 697.8(\varepsilon_1+\varepsilon_2)^2 - 39.6(\varepsilon_1+\varepsilon_2) + 0.5374, & \varepsilon_1+\varepsilon_2 < 0.021 \end{cases} \tag{7}$$

ν_{21} can be obtained from Eq.(2) once we have the values of E_1, E_2 and ν_{12}. Now a user material subroutine corresponding to these curve-fitting equations and the constitutive equation is designed and integrated to the ABAQUS input file.

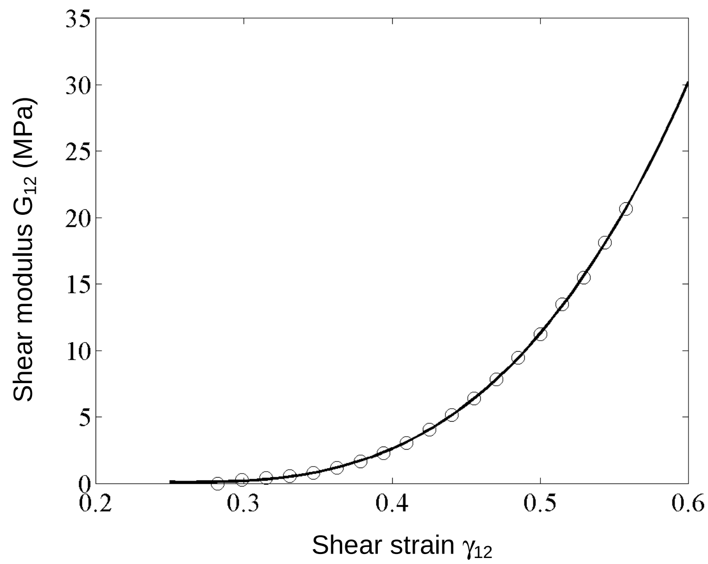

FIGURE 5. Shear modulus G_{12} of the composite.

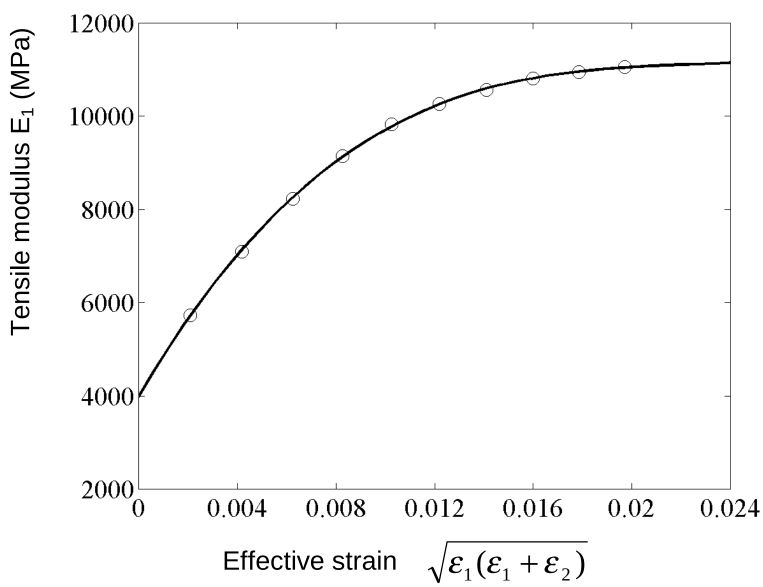

FIGURE 6. Tensile modulus E_1 of the composite.

VALIDATION OF THE MATERIAL PROPERTY MODEL

The previously obtained curve-fitting equations for material elastic constants are validated by a 30-degree bias trellising test and a bi-axial test. The reaction force versus displacement curves obtained from the unit cell and the single shell element are compared in Figs 8 and 9. A good agreement is obtained between the results from the unit-cell and the shell element, as shown in these figures.

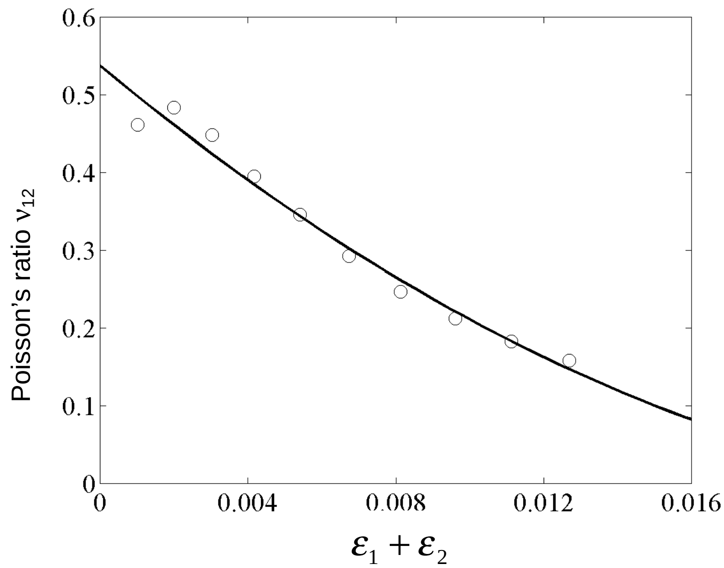

FIGURE 7. Poisson's ration ν_{12} of the composite

Now we can summarize the general procedure for predicting the effective material properties of textile composites:

1. build the unit cell for the fiber yarn.
2. apply the homogenization method to obtain the elastic constants of the fiber yarn.
3. build the unit cell for the textile composite.
4. characterize the behavior of force versus displacement by numerical trellising and uni-axial extension test on the unit cell via FEM analysis.
5. obtain the effective elastic moduli of the textile composite by correlating the force versus displacement curves of the unit cell and a shell element with the same size as the unit cell.
6. obtain curve-fitting equations for the effective elastic modul by choosing variables

CONCLUSIONS

A novel procedure is presented in this paper for predicting the effective nonlinear elastic moduli of textile composites by using the finite element formulation and the homogenization method. The procedure is exemplified on a plain weave composite by building a unit cell to represent the characteristic periodic pattern of the composite. The effective elastic moduli can be obtained from the trellising and uni-axial extension tests on the unit cell. Comparison with 30-degree bias trellising test and bi-axial extension test results validates the effectiveness of this procedure. The procedure can be easily extended to other textile composites by building the corresponding unit cell.

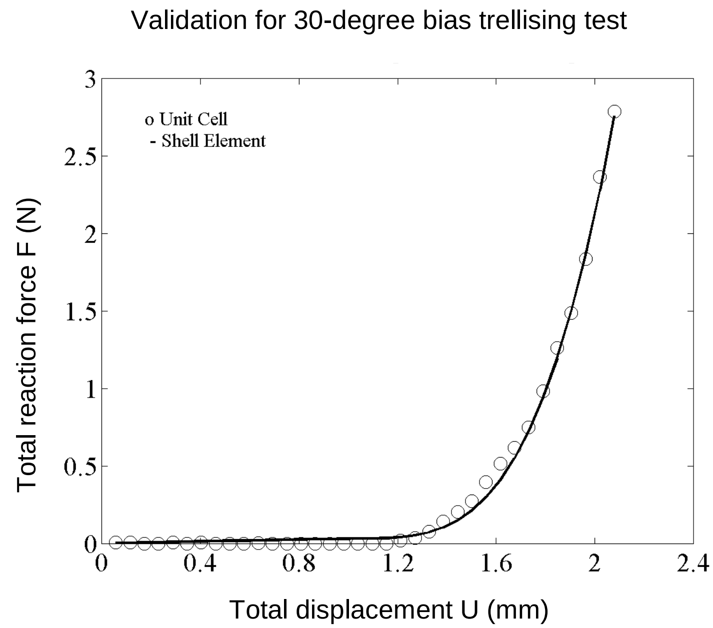

FIGURE 8. Total reaction force vs. total displacement

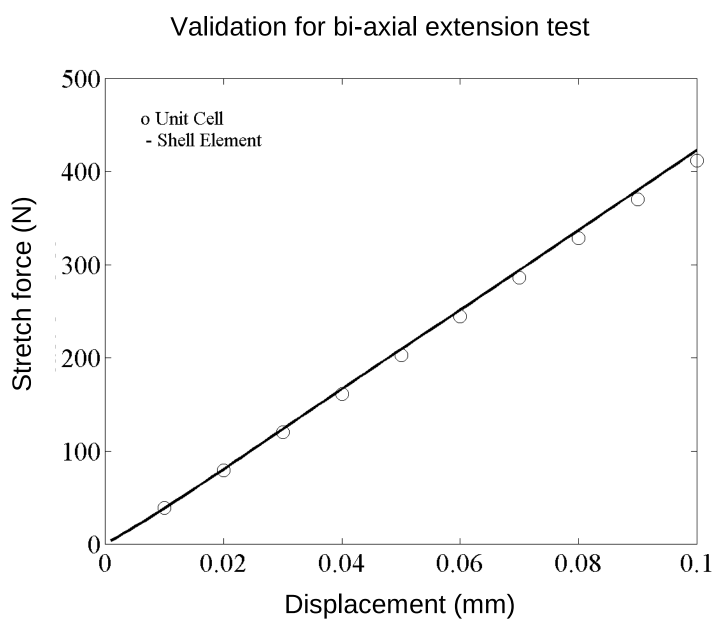

FIGURE 9. Load vs. displacement for biaxial extension

REFREENCES

1. A. Dasgupta and R. K. Agarwal. 1992. “Orthotropic Thermal Conductivity of Plain-weave Fabric Composite Using a Homogenization Technique,” *Journal of Composite Materials*, 26(18): 2736-2758.

2. J. Whitecomb, G. Kondaguta, and K. Woo. 1995. "Boundary Effects in Woven Composite," *Journal of Composite Materials*, 29(4): 508-524.

3. P. Boisse, A. Cherouat, J. C. Gelin, and H. Sabhi. 1995. "Experimental Study and Finite Element Simulation of a Glass Fiber Fabric Shaping Process," *Polymer Composites*, 16(1): 83-95.

4. P. Boisse, M. Borr, K. Buet, and A. Cherouat. 1997. "Finite Element Simulations of Textile Composite Forming Including the Biaxial Fabric Behavior," *Composites Part B-Engineering*, 28(4): 453-464.

5. A. Gasser, P. Boisse, and S. Hanklar. 2000. "Mechanical Behavior of Dry Fabric Reinforcements. 3D Simulations Versus Biaxial Tests," *Computational Materials Science*, 17: 7-19.

6. P. K. Liaw, N. Yu, D. K. Hsu, N. Miriyala, V. Saini, L. L. Snead, C. J. McHargue, and R. A. Lowden. 1995. "Moduli determination of continuous fiber ceramic composites (CFCCs)," *Journal of Nuclear Materials*, 219: 93-100.

7. S. W. Hsiao and N. Kikuchi. 1997. "Numerical Analysis of Deep Drawing Process for Thermoplastic Composite Laminates," *Journal of Engineering Materials and Technology*, 119: 314-318.

8. G. Weissenbach, L. Limmer, and D. Brown. 1997. "Representation of Local Stiffness Variation in Textile Composites," *Polymers & Polymer Composites*, 5(2): 95-101.

9. C. Wang and C. T. Sun. 1997. "Experimental Characterization of Constitutive Models for PEEK Thermoplastic Composite at Heating Stage During Forming," *Journal of Composite Materials*, 31: 1480-1506.

10. S. P. Ng, P. C. Tse, and K. J. Lau. 1998. "Numerical and Experimental Determination of In-plane Elastic Properties of 2/2 Twill Weave Fabric Composites," *Composites Part B*, 29: 735-744.

11. Y. M. Yi, S. H. Park, and S. K. Youn. 1998. "Asymptotic Homogenization of Viscoelastic Composites with Periodic Microstructures," *International Journal of Solids and Structures*, 35: 2039-2055.

12. X. Q. Peng and J. Cao. 1999. "Material Characterization in Forming Structural Composites," to be appeared in the Proceedings of the 100 Year Anniversary of Polytechnic University.

13. T. M. McBride and J. Chen. 1997. "Unit-cell Geometry in Plain-weave Fabrics During Shear Deformation," *Composites Science and Technology*, 57:345-351.

14. A. G. Prodromou and J. Chen. 1997. "On the Relationship Between Shear Angle and Wrinkling of Textile Composite Preforms," *Composites Part A*, 28: 491-503.