A STUDY ON THE STRESS DISTRIBUTION IN COIL WRAPPING AND ITS EFFECT ON FINAL COIL DEFORMATION

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ABSTRACT

Excessive coil deformation can hamper the normal operation of coil handling and bring difficulties in mass production. Experiences and previous researches have identified four critical factors: i.e., radial stiffness of coil material, winding tension, stiffness of core which supports the coil, and lubrication. In this paper, we advance our understanding of coil deformation by developing both linear and nonlinear models to predict the stress distribution in a coil during/after winding process and using a multi-layer equivalent model to study the coil deformation under gravity load. Using the proposed framework, the contribution of each factor to the coil deformation can be quantified and therefore provide scientific assistance in the engineering decision making process.

I. INTRODUCTION

In the production of sheet coils, the sheet is continuously wrapped onto a mandrel in the winding process. The stresses in the finished coil are not uniform due to the nature of the incremental wrapping process. The coil shape may change significantly under gravity load after being removed from the mandrel. This is referred to as the "soft coil" problem in industry, as shown in Fig.1. To ease this problem, a solid inner ring made of either metal or composite material is used to stiffen the coil. The factors influencing the coil behavior include: sheet properties, winding tension, lubrication and core stiffness [Becker, 1997].

In our previous study [Li and Cao, 2001], an equivalent model was used to model the coil deformation under gravity load and predict the core stiffness needed to prevent the soft coil problem. However, residual stresses in the coil after the winding process were not considered and the contributions of each factor to the final deformation were not quantified, nor in other literatures available to the authors.

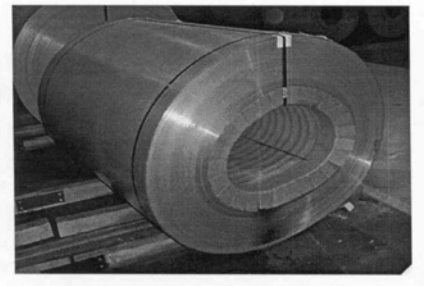


FIGURE 1 "SOFT COIL ".

Altman [1968] proposed one of the earliest analytical models to calculate the internal stresses of a wound roll. Hakiel [1987] presented a nonlinear model by assuming that the radial Young's modulus of the coil varies in the radial direction in the form of a polynomial function. The same concept was adopted in Benson [1995] and he showed that the radial and interlayer behaviors can be uncoupled. There are also other studies based on a hyper-elastic model or a visco-elastic model [Yagoda 1980, Willett 1988, Zabaras 1994, Qualls 1997, which are used for analyzing the wrapping magnetic tape and paper coils. Yuen and Cozijnsen [1996] studied steel coils by using a material model considering the interlayer gap effect and applied the results to analyze the tension profile that prevents the buckle problem [Yuen 2000].

In above studies, the following assumptions were adopted:

1) The coil is considered as an assembly of pre-stressed rings shrank-fit together.
2) The coil retains its axisymmetric shape during winding and after removal from the mandrel.

3) The radial properties of the coil are different from those of the sheet and are considered either uniform in a linear model or follow a certain function in a non-linear model.
4) Bending is not taken into account because the coil radius is much larger than the sheet thickness.

In the present study, material properties are determined by a compression test of the multi-sheets stacked together. Then the wrapping process is formulated using series of linear/nonlinear differential equations to calculate the stresses after a new wrap is added. The stress distribution obtained after wrapping and mandrel removal is then used to evaluate the radial stiffness distribution of the coil. A multi-layer equivalent finite element model is developed to account for the non-uniformity of the stiffness. With these, the contribution of each factor can be analyzed.

II. MATERIAL MODEL

In general, materials such as paper, tape and metal sheets posses some anisotropic behavior, i.e. the properties in the in-plane directions and the out-of-plane direction are different. Within a wound coil, the properties in the radial direction and the tangential direction are also different due to the interlayer effect in addition to the material anisotropy. In this paper, we are particularly interested in the laminate sheet—two steel skins with a thin composite core in between. This kind of material is widely used in applications for the purpose of vibration and noise reduction.

Due to the composite layer, the difference between the out-of-plane property and the in-plane property of this material is much larger than that of solid steel sheets. The radial stiffness of a laminate coil is much lower and leads to the soft-coil problem as mentioned in the introduction.

To accurately characterize the radial behavior of a coil, a multiplayer compression test (Fig.2) of laminate sheets is used to characterize the radial property of the coil.

The stress-strain relationship can be described by assuming that the radial stiffness is a function of pressure. A polynomial function by can be used as follows:

$$E_r = \frac{dp}{d\varepsilon_r} = c_1 + c_2 p + c_3 p^2 \qquad (1)$$

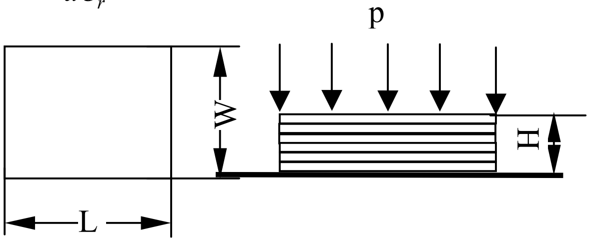


FIGURE.2 SCHEMATIC OF THE COMPRESSION TEST OF A STACK OF SHEET

where p is the compression stress, ε_r is the compressive strain, and c_1, c_2, c_3 are constants which should be determined by fitting the experimental curve using (1). Higher order polynomial or other function forms may be used to fit a particular material. Fig.3 shows several common models.

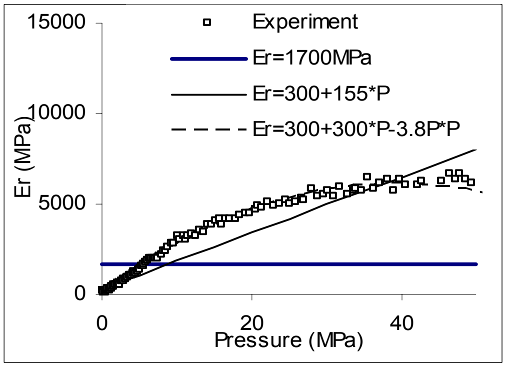


FIGURE 3 DIFFERENT MATERIAL MODELS.

The tangential properties of the coil are considered the same as laminate sheet and will not change with stress.

III. BASIC FORMULATION

The coil is viewed as an axisymmetric body as shown in *Fig.4*. The longitudinal deformation is ignored, i.e. plane strain condition is assumed. Before wrapping a new layer, a coil with inner radius r_i and external radius r_o must satisfy the equilibrium equation:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (2)$$

The subscript 'i' and 'o' denote the inner and external surface respectively. When a new wrap is added to the coil, the stresses in the coil will change to reach a new equilibrium state. It is obvious to know that the changes in stresses also satisfy the equation:

$$\frac{\partial \Delta\sigma_r}{\partial r} + \frac{\Delta\sigma_r - \Delta\sigma_\theta}{r} = 0 \qquad (3)$$

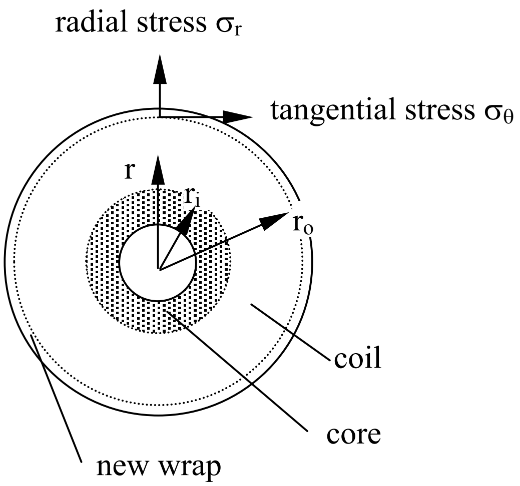


FIGURE.4 STRESS AND GEOMETRICAL NOTATIONS OF A COIL.

Let u denote the radial displacement caused by the new wrap at any position in the existing coil, ε_r and ε_θ denote the corresponding strain increments in the radial and tangential direction. The strain-displacement relation can be shown in (4).

$$\begin{cases} \varepsilon_r = \dfrac{du}{dr} \\ \varepsilon_\theta = \dfrac{u}{r} \end{cases} \qquad (4)$$

As described in Benson (1995), the radial and tangential deformation behavior can be considered independent, i.e. the Poisson's ratios are zero, the constitutive equations thus reduce to the simple form:

$$\begin{cases} \varepsilon_r = \dfrac{\Delta\sigma_r}{E_r} \\ \varepsilon_\theta = \dfrac{\Delta\sigma_\theta}{E_\theta} \end{cases} \qquad (5)$$

After the new wrap is shrunk-fit to the coil, the amount of shrinkage at both the inner and external surfaces of the coil must satisfy the boundary conditions described below.

During the wrapping process, a new layer with an initial tensile stress σ_0 is added to the existing coil with an external radius r_e. The inner radius of the new layer at zero stress state r_0 can be calculated as:

$$r_0 = r_e e^{-\frac{\sigma_0}{E_\theta}} \qquad (6)$$

After shrunk-fit, the external radius of the existing coil is reduced to r_e' from r_e., This introduces a new tangential stress:

$$\sigma_t^{n+1} = E_\theta \ln \frac{r_e'}{r_0} \qquad (7)$$

and a radial pressure at the external surface of the existing coil:

$$\sigma_r^{n+1} = \frac{t}{r_e'} \sigma_t^{n+1} \qquad (8)$$

where t is the sheet thickness. Boundary conditions at the external surface of the existing coil are:

$$p_e = \sigma_r^{n+1}, u_e = r_e' - r_e \qquad (9)$$

At the interface between the coil and core, the radial pressure p_i of the coil must balance the reaction pressure generated by the core. During the wrapping process, the core is considered as an elastic foundation to the coil. The relationship between the pressure p_c and the radial displacement u_i can be obtained using the thick cylinder formula:

$$p_c = \frac{E_c\left(\frac{1+\nu_c}{1-\nu_c} r_{ce}^2 + r_{ci}^2\right)}{(1+\nu_c)(r_{ce}^2 - r_{ci}^2)} \frac{u_i}{r_{ce}} = k_w u_i \qquad (10)$$

where k_w is defined as the core stiffness. E_c and ν_c are the Young's modulus and Poisson's ratio, r_{c1}, r_{c2} are the inner and external radii of the core.

3.1 Linear model

If the radial modulus of the coil is assumed to be constant, i.e. it does not change with applied pressure, the equilibrium equation can be reduced to the following simple form by inserting Eqs. (4) and (5) into (3):

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{E_\theta}{E_r}\frac{u}{r^2} = 0 \qquad (11)$$

The general solution of the above differential equation is:

$$u = C_1 r^{\alpha} + C_2 r^{-\alpha} \qquad (12)$$

where $\alpha^2 = E_\theta / E_r$

Using the boundary conditions:

$$\begin{cases} \sigma_r = p_e \\ u = u_e \end{cases} \quad \text{at } r = r_e$$

The constants in (12) can be determined as:

$$\begin{cases} C_1 = \dfrac{u_e + \dfrac{p_e}{\alpha E_r} r_e}{2 r_e^{\alpha}} \\ C_2 = u_2 r_e^{\alpha} - C_1 r_e^{2\alpha} \end{cases} \qquad (13)$$

The displacement u_i and pressure p_i at the inner surface of the coil can be obtained as:

$$\begin{cases} u_i = C_1 r_i^{\alpha} + C_2 r_i^{-\alpha} \\ p_i = \alpha E_r (C_1 r_i^{\alpha-1} - C_2 r_i^{-\alpha-1}) \end{cases} \qquad (14)$$

Using the equilibrium equation solution (12) and the boundary conditions expressed in Eqs. (6-10) the displacement and stress increments caused by a new wrap can be calculated. The new geometry and stresses in the coil can be updated. The final stresses and geometry of a coil can be obtained by repeating the above process for each new wrap until the desired wrap number is reached.

3.2 Nonlinear model

As the compression experiment shows, the radial stress-strain relationship is nonlinear. To consider this effect, a non-linear relationship between the radial stiffness and the pressure should be used. Assuming that the incremental stresses are small, the stress increments caused by a new wrap can be obtained as:

$$\begin{cases} \Delta\sigma_r = E_r \dfrac{du}{dr} \\ \Delta\sigma_\theta = E_\theta \dfrac{u}{r} \end{cases} \qquad (15)$$

Substitute (15) into (3), the control differential equation becomes:

$$\frac{du^2}{dr^2} + \left[\frac{dE_r}{dr} \frac{1}{E_r} + \frac{1}{r} \right] \frac{du}{dr} - \frac{E_\theta}{E_r} \frac{u}{r^2} = 0 \qquad (16)$$

where E_r is the radial stiffness modulus of the existing coil and is a function of radius r.

Unlike the linear model, equation (16) is a nonlinear second order differential equation and no explicit solution can be obtained. A numerical method has to be used to obtain the displacement and stress fields using boundary conditions described in the linear model.

3.3 Residual stress calculation after unloading

In the linear case, the stresses after the mandrel being removed can be calculated by adding an equal pressure that is opposite to the stress at the inner surface after wrapping. However, in the nonlinear case, an incremental procedure must be used due to the non-linearity of stress-strain relation. A small value of opposite pressure to the inner surface is added until the pressure at the inner surface is zero. The stresses in the coil are updated in each step.

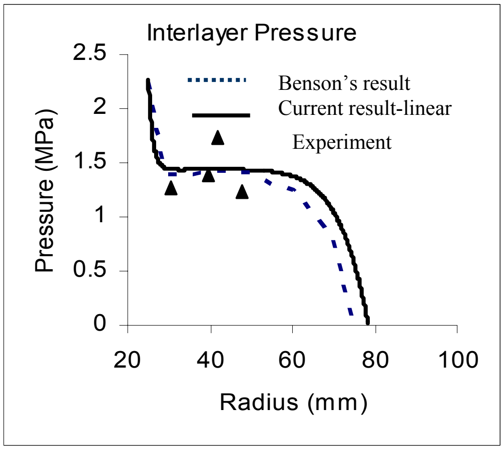
3.4 Validation of the model

To test the effectiveness of the current approach, the example in Benson [1995] is used to compare the results. The nonlinear model he used was:

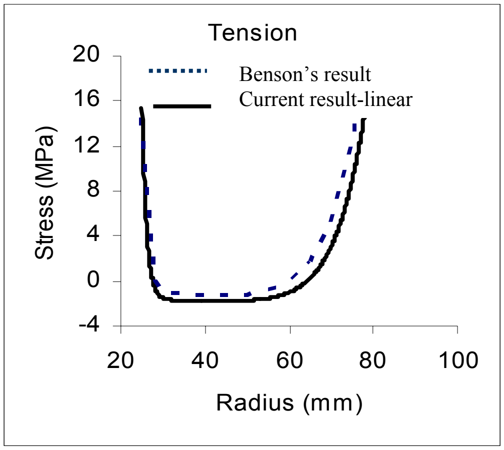
$$E_r = \alpha\beta \left(1 + \frac{p}{\alpha} \right)^{(\beta+1)/\beta} \qquad (17)$$

where α=0.0479Mpa and β=26.3.

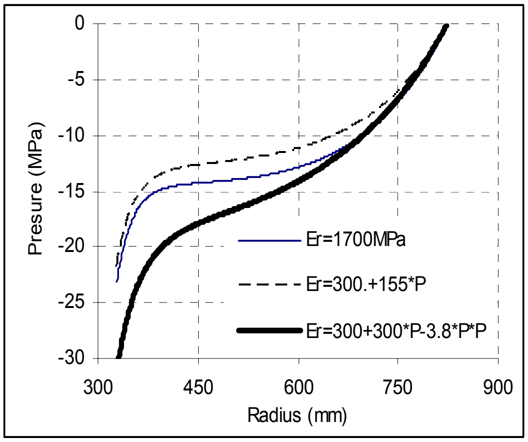
Figure 5 is the comparison of the 750 wraps paper coil. The data are obtained by digitizing the curves in his paper. The value E_r=*20MPa* is used in the analysis. The range of Er in (17) is 1.28~68MPa. It can be seen clearly that the results are very similar even though only the linear model is used.



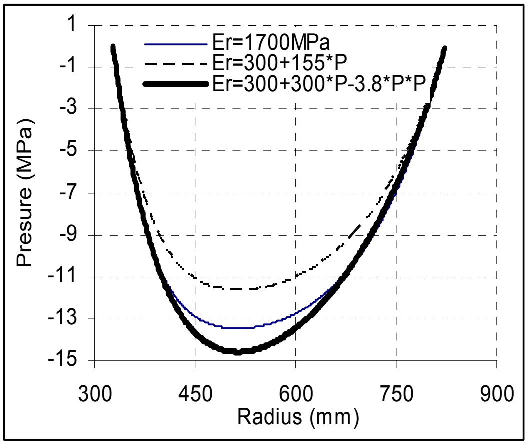
(a) Radial stress



(b) Tangential stress
FIGURE 5 COMPUTED RESULTS OF INTERLAYER PRESSURE AND TENSION.



(a) Radial stress after wrapping



(b) Radial stress after mandrel removal
FIGURE 6 COMPARISON BETWEEN LINEAR AND NON-LINEAR MODELS.

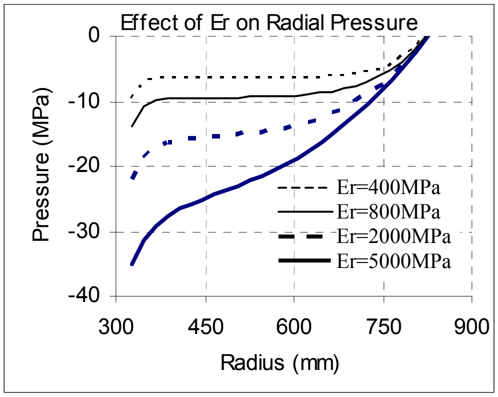
Figure 6 shows the radial stress distributions using the linear and two non-linear models for the laminate coil. The Er-p relationship is shown in Fig.3. The calculated tangential stresses are very close and are not shown here. The radial stresses after wrapping show a relatively large difference while the residual radial stress are much closer. If a right value of *Er* is chosen in the linear model, very similar results can be obtained. The advantage of the nonlinear model is that test data can be directly used without guessing what *Er* value should be used.

In the following section, only linear model is used to clearly demonstrate the effects of the different parameters on the stress distribution. The nonlinear model will be used to calculate the stresses final coil deformation under gravity load in real application cases.

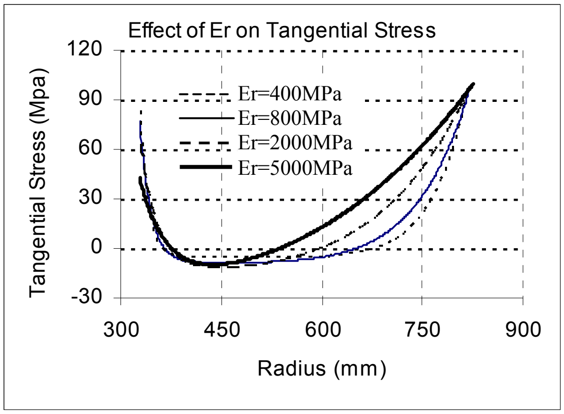
IV. RESULTS OF STRESS ANALYSIS

The approach developed above provides us a means to examine the effects of tension and material properties on the stress distribution during wrapping and after unloading. In this section, we will study the effects of radial stiffness, winding tension and core stiffness on the stress distributions in a wound laminate coil.

Note that, in general, a higher residual pressure corresponds to more resistance to slippage between the layers and therefore, is more desirable for preventing the "soft coil" problem. Similarly, a higher residual tangential stress also provides more resistance to the coil deformation. However, a high value of the ratio of this stress to material stiffness can also lead to potential buckling problems as pointed out by Yuan [2000].

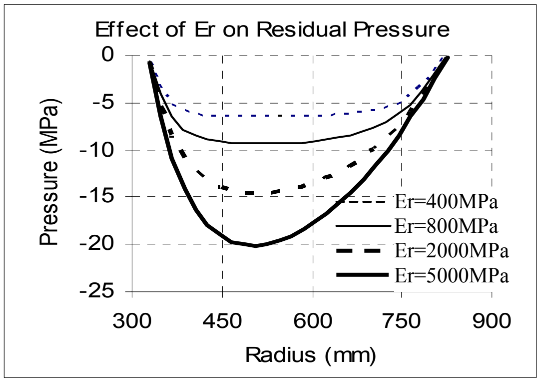


(a) radial pressure

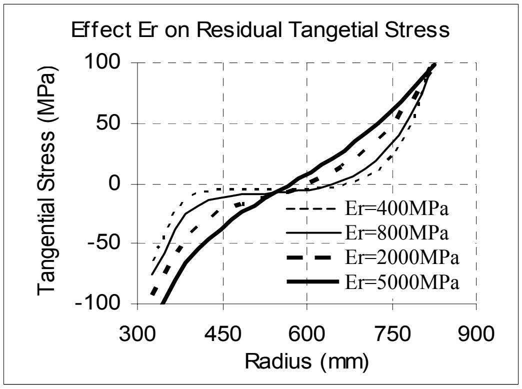


(b) Tangential tension

FIG. 7 EFFECTS OF *Er* ON STRESS DISTRIBUTION AFTER WRAPPING.



(a) Residual Pressure



(b) Residual tangential stress

FIGURE 8 EFFECTS OF *Er* ON STRESS DISTRIBUTION AFTER MANDREL REMOVAL.

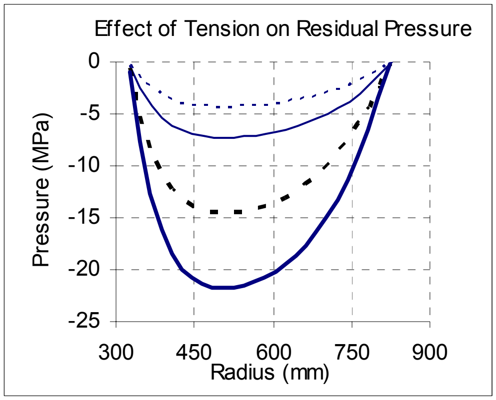
All the analyses are performed for a 500 wrap coil of 1mm thick laminate sheet. The material parameters used are listed in the following:

Core: E_c=4500MPa, v_c=0.3,
R_{ci}=303.5mm, r_{ce}=328.6mm
Sheet: t=1mm, E_θ=106500MPa

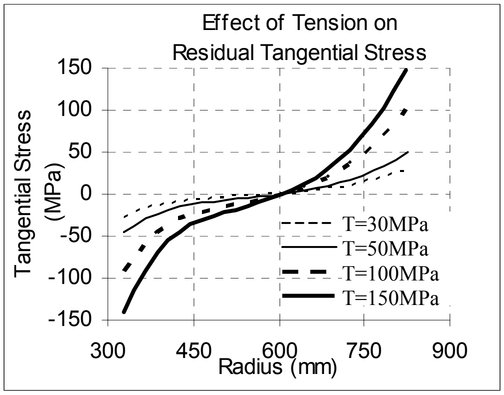
Figure 7 shows the effects of radial modulus E_r on the stress distributions after wrapping. The tension used is 100MPa. It can be seen that the radial pressure increases with an increase in E_r. The tangential stress distribution changes drastically while the maximum and minimum values do not vary that much. The radial pressure increases with E_r.

Figure 8 shows the effect of *Er* on the residual stresses after the mandrel removal. The residual inter-layer pressure increases with *Er* and the distribution curve also becomes sharper. As *Er* increases from 400MPa to 5000MPa, the stress at the inner surface normalized by Er reduces from 0.17 to 0.02. This observation indicates that an increase in Er is desirable for the prevention of both "soft coil" and buckling problems.

The effect of wrapping tension is shown in *Fig.9*. The increase in wrapping tension causes an increase in the interlayer pressure but also causes an increase in the compressive tangential stress at the inner surface. This means that an increase in tension will help to retain a high radial pressure and prevent the "soft coil" problem, but the increase in the compressive residual tangential stress may cause buckling problems.

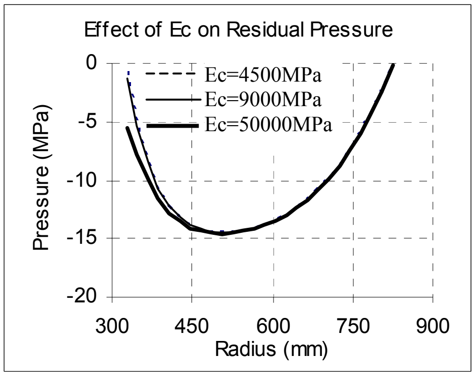


(a) Residual pressure

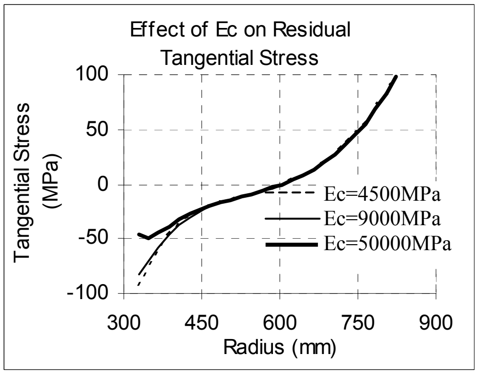


(b) Residual tangential stress

FIGURE 9 EFFECT OF TENSION ON STRESS DISTRIBUTION AFTER MANDREL REMOVAL.



(a) Residual pressure



(b) Residual tangential stress

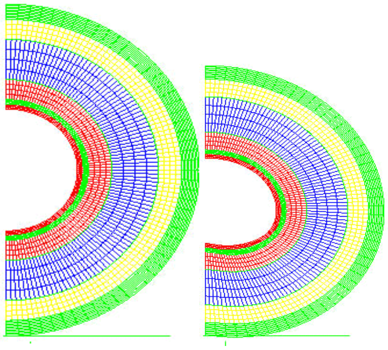
FIGURE.10 EFFECT E_c ON RESIDUAL STRESS.

The core stiffness is very important for the coil behavior. *Fig.10* shows the effect of core modulus on the residual stress distribution. (The effect of core stiffness on the stress distribution before mandrel removal is small and is not shown here). The core modulus has little effect on the stress at the outer part of the coil. The major effect of an increasing core modulus is that the radial pressure at the inner side is increased and the compressive tangential stress is decreased. This increases the coil's stiffness to resist gravity and also helps to prevent the buckling problem due to the excessive compressive tangential stress at the bore.

V. ANALYSIS OF COIL DEFORMATION

The coil is under gravity load after being removed from the mandrel. The axisymmetric model developed for the stress distribution cannot be used to calculate the deformation under gravity load due to the asymmetric nature involved in this problem.

Here, we will use a finite element model to perform the analysis. The challenge in establishing a finite element is how to effectively and accurately capture the fundamental mechanisms. As a coil may contain hundreds or thousands of layers, we propose a multi-layer model in which the equivalent shear modulus of each layer is set to a different value to model the non-uniformity of the radial property, as shown in Fig.11. The layers are designed according to the residual pressure stress distribution to capture the non-uniformity.



(a) multi-layer model (b) deformed shape

FIGURE.11 THE MULTIPLAYER MODEL.

Each layer is modeled by plane strain solid elements. The layer to layer interaction is modeled using contact surfaces and friction can also be taken into account by using Columbus friction law. The internal stresses obtained from the previous axisymmetric model are used to evaluate the equivalent shear modulus in each layer. The exact relationship between the equivalent shear modulus and residual pressure and friction can be studied by using experimental data of coil deformation under different winding tension and lubrication conditions. This will be explored in detail in future studies.

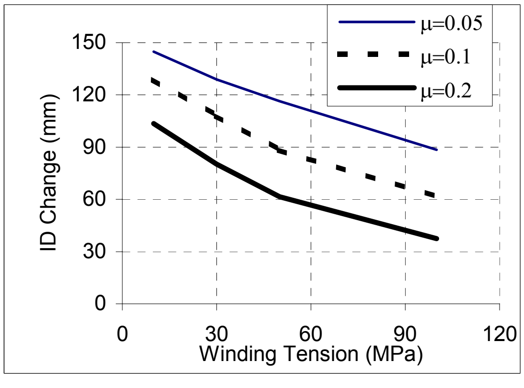


FIGURE.12 EFFECTS OF WINDING TENSION AND FRICTION ON ID CHANGE.

The coil deformation under gravity load is analyzed using the commercial finite element package, ABAQUS/Standard. The effects of tension and friction coefficient are studied and the results of a test problem are shown in *Fig.12*. The shear modulus is simply taken as the friction stress between the neighboring wraps. It can be seen that a larger friction coefficient and winding tension will reduce the inner diameter (ID) changes of the coil and will help to prevent the "soft coil" problem.

VI. SUMMARY

A method has been developed to compute the internal stresses of a coil during winding process and after mandrel removal. As coil materials may have different material behaviors, a linear and a nonlinear model are developed. Here, we present both models so that the readers can decide which one is more suitable to their applications. The nonlinear model can usually reflect the actual material behavior while the latter one has its own advantage as having a closed form solution. It has been shown that for a certain value used in the linear model, the predicted results are quite similar to that obtained from the nonlinear models (Fig.5). Of course, the question on what that value should be needs further investigations.

Parametric studies using the developed model are conducted. With the understanding that a high residual pressure leads to more shear resistance in coil deformation, we conclude that the occurrence of the "soft-coil" problem can be reduced when the radial stiffness of the coil, the tension in winding and the stiffness of the core material are increased. Caution has to be taken when using high tension, as tension will result in a high ratio of inner residual tangential stress to the material stiffness which may lead to possible buckling.

We further our analysis by using a multi-layer Finite Element Model to calculate the change of a coil's inner diameter under gravity load. The number of layers in the FEM model is significant lower (one or two orders smaller) than that of the coil, which makes solution very efficient. On the other hand, the number of layers is determined by the residual stress distribution of the coil after being removed from the mandrel to account for the non-uniformity of the radial stiffness distribution, which provides us a means to capture the right physics in the coil deformation problem. The equivalent shear modulus of each layer is determined by the residual stress and the interface behavior between the layers.

The proposed models and approach allow us to quantitatively study the contributions of each four critical factors (i.e., material stiffness, tension, lubrication and core stiffness) to the coil deformation. The methodology can be used to explicitly answer questions such as what minimum core stiffness is needed to support the coil, what is the maximum number of layers that the core can support, what process parameters we can adjust or what lubricant we can use, etc. Further work is needed to compare the numerical results with field data to improve the model and develop specific strategies for different applications.

ACKNOWLEDGEMENT

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REFERENCES:

Altman, H.C., (1968), Formulas for computing the stresses in center-wound rolls, *Journal of the Technical Association of Paper and Pulp Industry*, vol.51, No.4, pp 176-179

Benson, R.C., (1995), Nonlinear wound roll model allowing for large deformation, Proceedings of the 1995 *ASME International Mechanical Engineering Congress & Exposition*, San Francisco, CA, USA

B.J. Becker, (1997), A Systems Approach to reducing Winding defects at Alcoa-Warrick operations, Proceedings of the International Conference on Web Handling, pp 102-114

Cozjinsen, M., Yuen, W.Y.D., (1996), Stresses in wound rolls, 2nd *Biennial Australian Engineering Mathematics Conference*, July, Sydney, Australia, pp 117-124.

Hakiel, Z., (1987) Nonlinear model for wound roll stresses, *Tappi J* v 70, n 5 , pp 113-117.

Li, S.P., Cao, J., (2001), Enhancement of coil stiffness using a cardboard core, ASME International Mechanical Engineering congress & Exposition, Nov., 2001, New York.

Qualls W.R., Good J.K., (1997), Orthotropic viscoelastic winding model including a nonlinear radial stiffness, *J Appl Mech Trans ASME* v 64 : 1 , pp 201-208.

Willett, M. S., Poesch, W. L., (1988), Determining the stress distributions in wound reels of magnetic tape using a nonlinear finite-difference approach, *J Appl Mech Trans ASME* v 55, n 2 , pp 365-371.

Yagoda, H. P., (1980), Resolution of a core problem in wound rolls, *J Appl Mech Trans ASME* v 47, n 4, p 847-854.

Yuen, W.Y.D., M. Cozjinsen, (2000), Optimum tension profiles to prevent coil collapses, *SEAISI Quarterly*, July, pp50-59.

Zabaras, N. Donaldson, E., (1994), Hypo-elastic model for computing the stresses in center-wound rolls of magnetic tape, *J Appl Mech Trans ASME,* v 61, n 2 , pp 290-295.